

# STAR FORMATION IN THE MULTIPHASE INTERSTELLAR MEDIUM OF GALAXIES

Chris McKee

Galaxy Formation Workshop

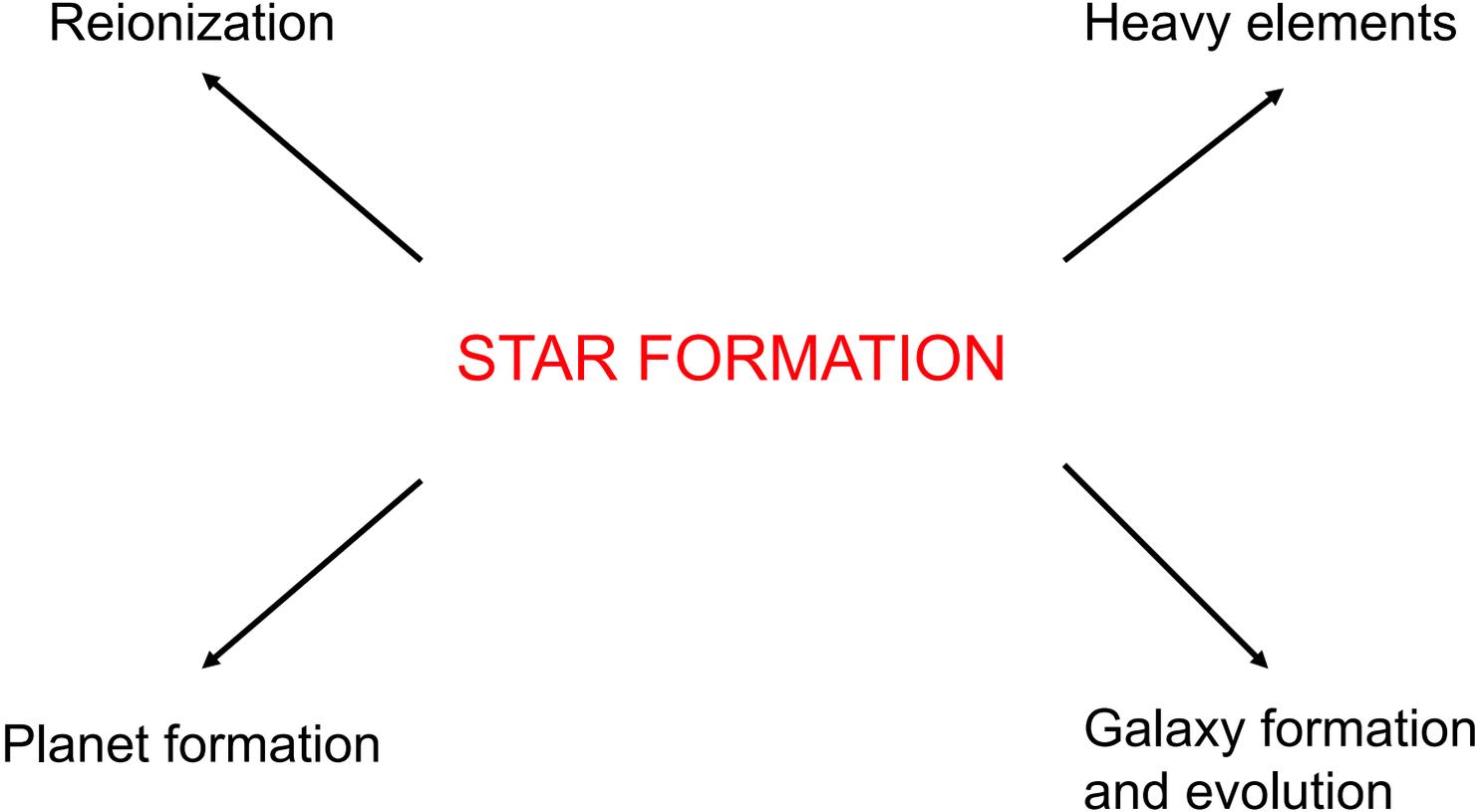
UCSC August 9, 2011

With:

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Eve Ostriker, & Jason Tumlinson

# STAR FORMATION IS AT THE NEXUS OF ASTROPHYSICS



Stars form in molecular gas ( $H_2$ ) because molecular gas is cold

Krumholz, Leroy & McKee (2011)

(See also Schaye 2004; Gnedin & Kravtsov 2011)

Calculate temperature as function of density  $n_H$  and column density  $N_H$  for different possible coolants; hatched regions not self-consistent.

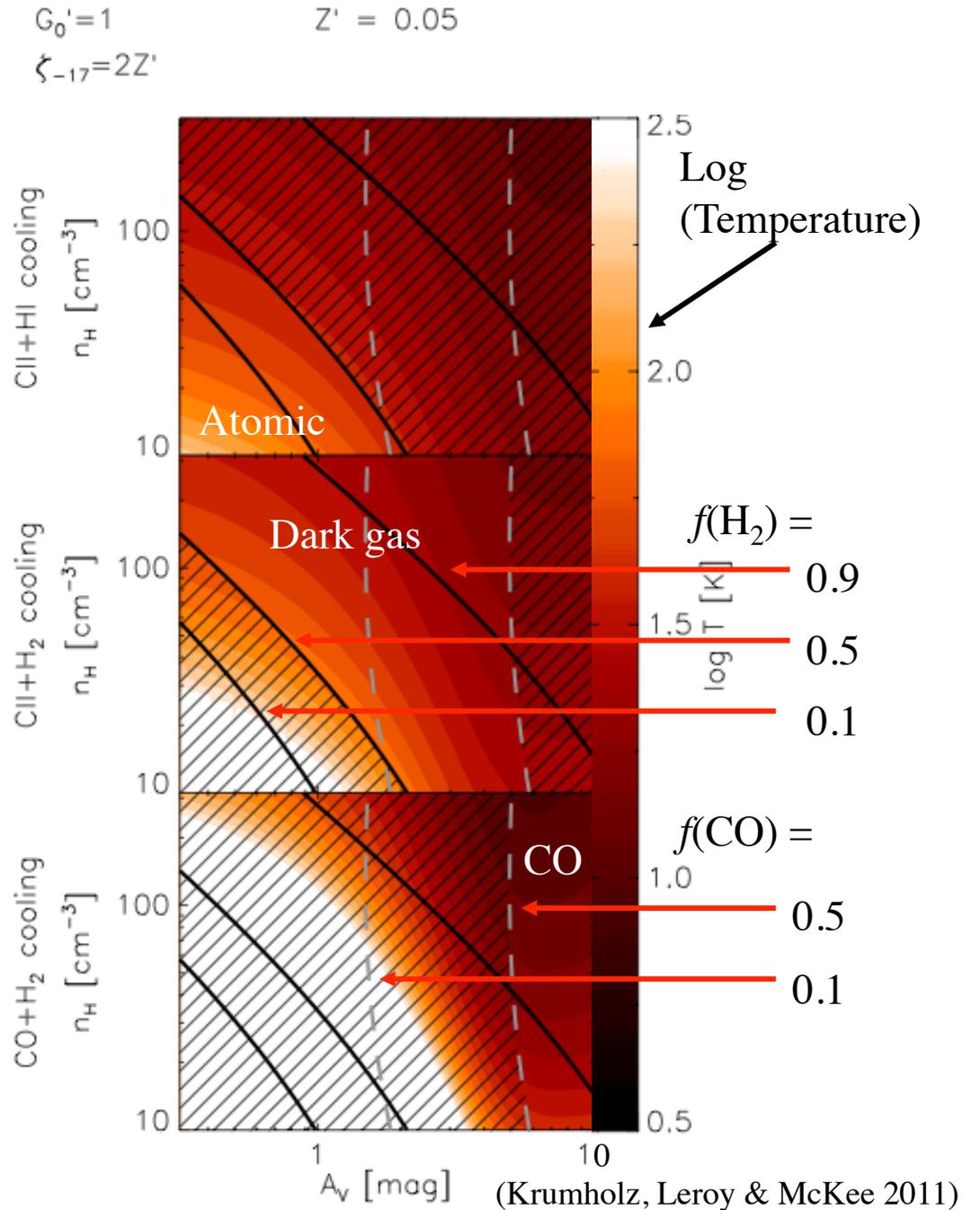
Contours of  $H_2$  concentration follow isotherms:

Heating and dissociation  $\propto \exp(-A_V)$

Cooling and formation  $\propto n^2$

Contours of CO concentration are at constant extinction

Hence cold gas is primarily  $H_2$  over wide range of density and column



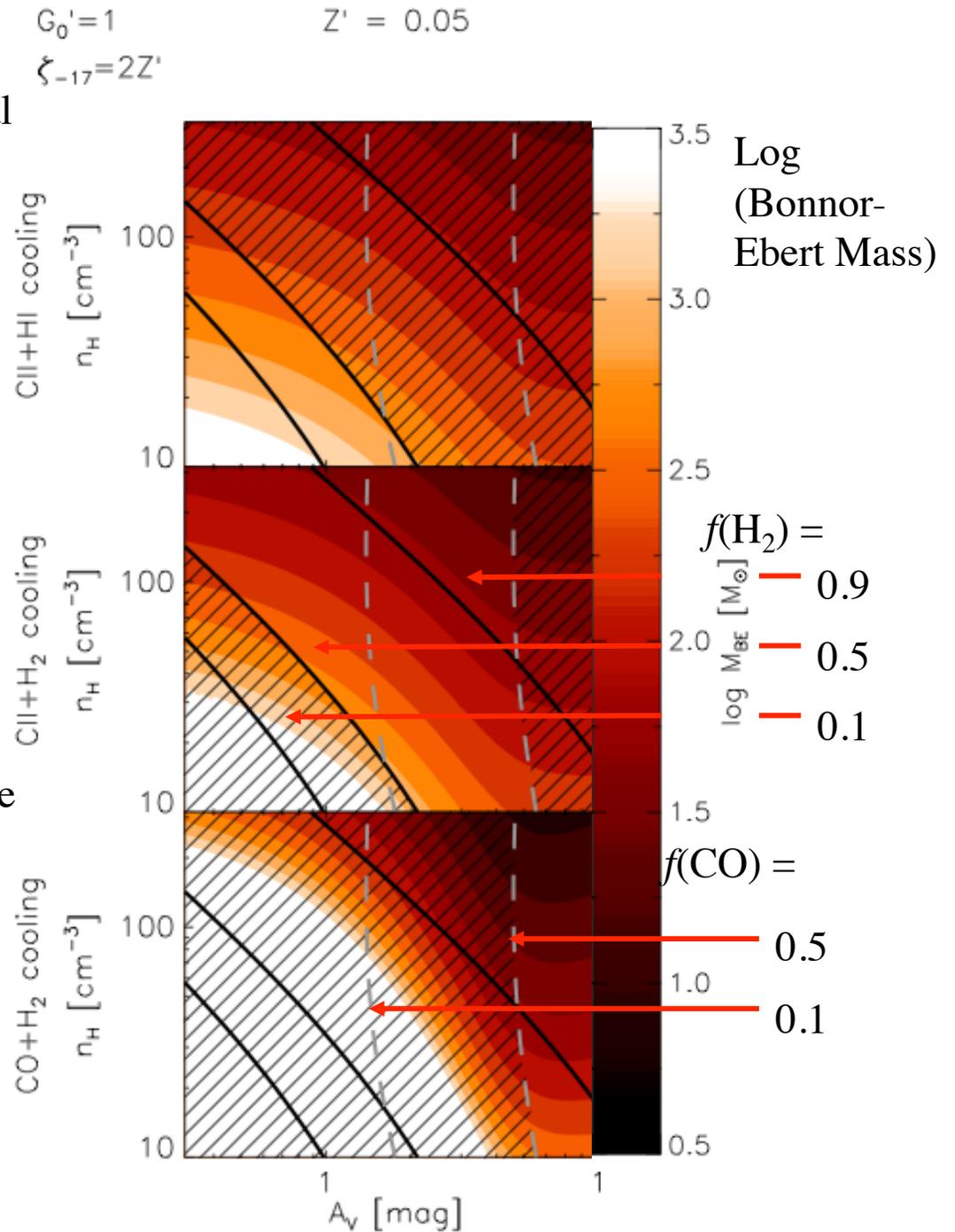
Maximum stable mass of an isothermal spherical cloud (Bonnor-Ebert mass):

$$M_{\text{BE}} = 1.18 c_s^3 / (G^3 \rho)^{1/2}$$

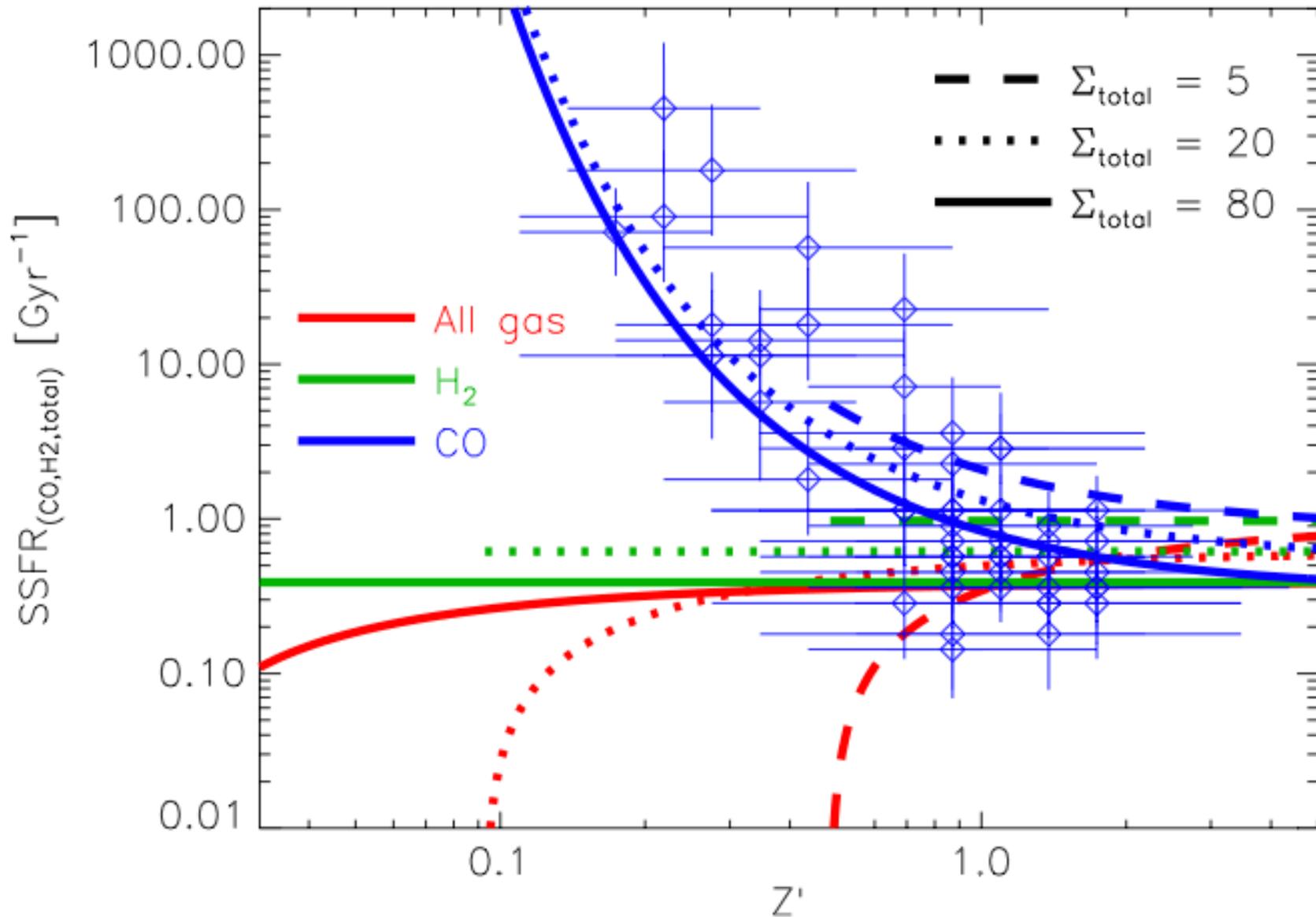
Strongly correlated with  $\text{H}_2$  fraction, not CO fraction

Atomic gas: stars cannot form since  $M_{\text{BE}} > \sim 1000 M_{\text{sun}}$

Molecular gas ( $\text{H}_2$ ): stars can form since  $M_{\text{BE}} < \sim 10 M_{\text{sun}}$



## Specific star formation rate as a function of metallicity



Observations show rapid rise of SSFR for CO as  $Z'$  decreases, consistent with theory

# A TALE OF TWO THEORIES: THE STAR FORMATION RATE IN A MULTIPHASE ISM

## I. Turbulence-Regulated Star Formation (Krumholz & McKee 2005; Krumholz, McKee & Tumlinson 2008, 2009a,b; McKee & Krumholz 2010)

- Based primarily on theory
  - General (includes starbursts, low metallicity), but approximate
- At low surface densities or metallicities,  $\text{SFR} \propto f(\text{H}_2)$ , the molecule fraction
  - Determined from physics of  $\text{H}_2$  molecule

## II. Regulation of Star Formation in a Two-Phase ISM in Galactic Disks

(E. Ostriker, McKee & Leroy 2010)

(Extension to low Z: Bolatto et al. 2011)

- Star formation rate per unit area:  $\Sigma_{\text{SFR}} = \Sigma_{\text{GBC}} / t_{\text{SF}}$ 
  - $\Sigma_{\text{GBC}}$  = surface density of gravitationally bound clouds, determined from hydrostatic equilibrium in 2-phase ISM, independent of  $\text{H}_2$ .
  - $t_{\text{SF}} = 2 \times 10^9 \text{ yr}$  = star formation time in molecular gas in local galaxies
- Determine  $\Sigma_{\text{GBC}}$  self-consistently in thermal and dynamical equilibrium
- Gives accurate radial profiles for SFR in disk galaxies in local universe

# Both Theories Are Based on HI in Two-Phase Equilibrium

(Wolfire, McKee, Hollenbach, & Tielens 2003)

Ionization: FUV, X-ray, C.R.

Heating: P.E., C.R., X-ray/EUV

Cooling: [CII], [OI], Ly $\alpha$ ,  
e<sup>-</sup> recombination

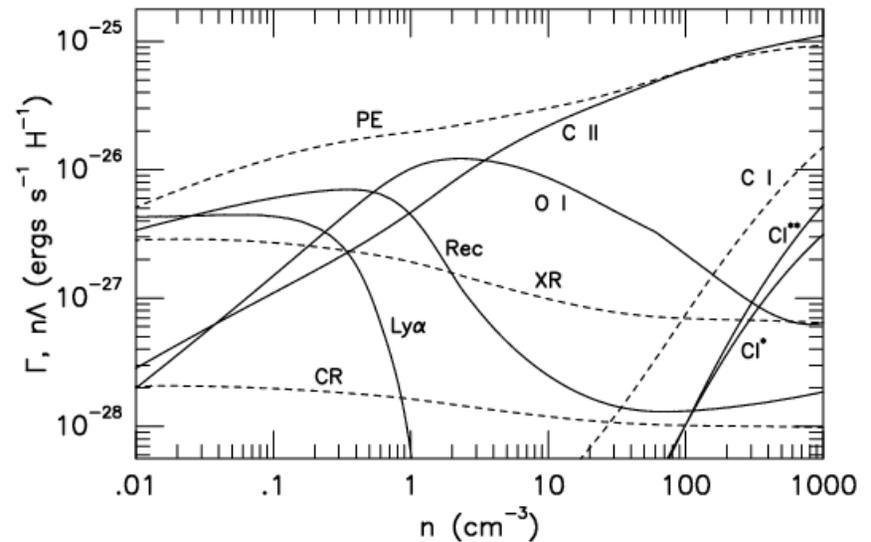
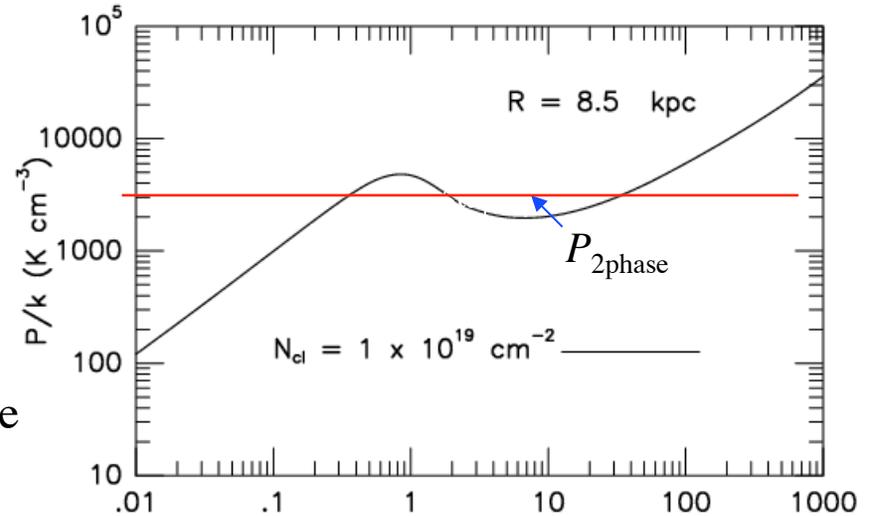
$$P_{2\text{phase}}/k \approx 3000 G_0'/\phi_D \quad \text{K cm}^{-3}$$

where  $G_0'$  = FUV radiation/local ISM value

$$\phi_D = (1 + 3.1 Z'^{0.365})/4.1$$

$Z'$  = metallicity relative to solar

$$n_{2\text{phase}} \approx 20 G_0'/\phi_D \quad \text{cm}^{-3}$$



# THEORY I: TURBULENCE-REGULATED STAR FORMATION

(Krumholz & McKee 2005)

The star formation rate per unit area is

$$\Sigma_{\text{SFR}} = d\Sigma_*/dt = (\epsilon_{\text{ff}}/t_{\text{ff}}) f_{\text{GMC}} \Sigma_{\text{g}}$$

where

$\epsilon_{\text{ff}}$  is the star formation rate per free-fall time in a Giant Molecular Cloud (GMC)

It is the fraction of the mass of a GMC converted into stars in one free-fall time---ie, it is a star formation efficiency.

$t_{\text{ff}} = (3\pi/32 G\rho)^{1/2} = 4 \times 10^7 v^{1/2} \text{ yr}$  is the gravitational free-fall time

$f_{\text{GMC}} \Sigma_{\text{g}}$  is the surface density of molecular gas in GMCs

Notes:

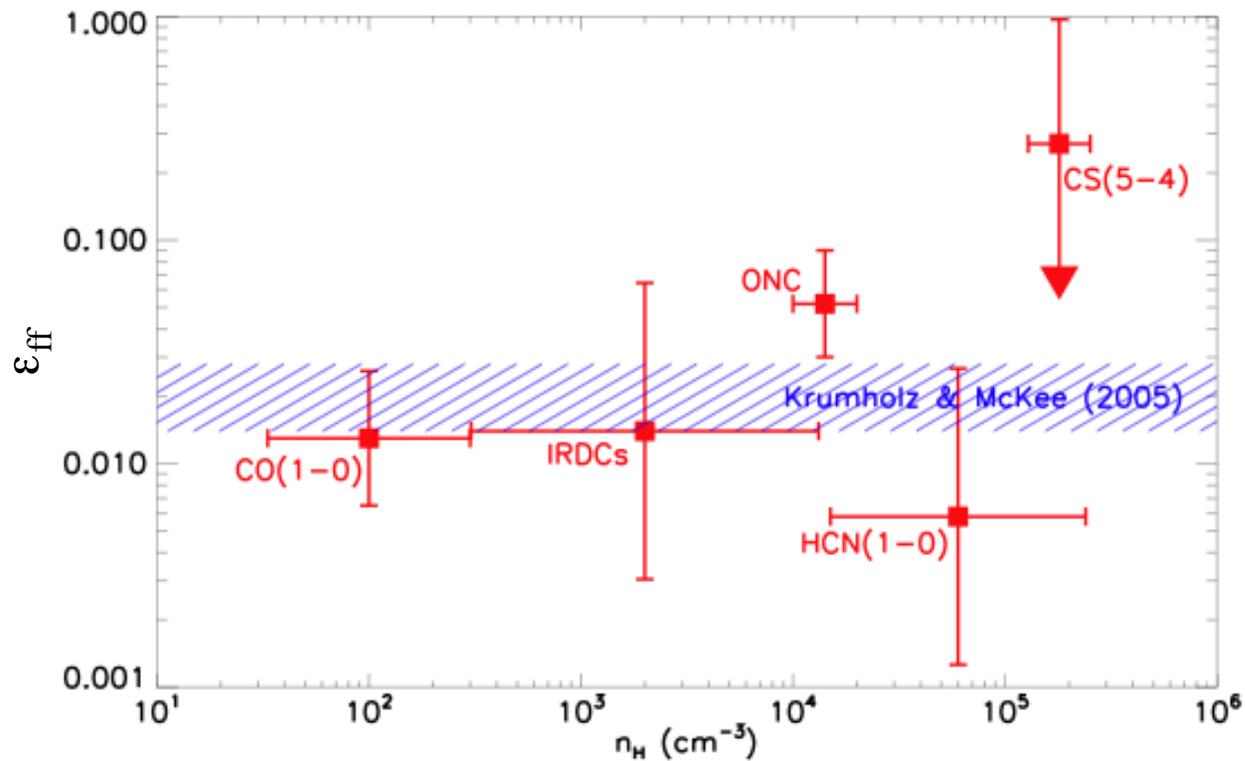
\*The basis of this equation is that stars form in gravitationally bound clouds of molecular gas -- GMCs

\* This equation clarifies the three quantities that determine the SFR: (1) the efficiency,  $\epsilon_{\text{ff}}$ ; (2) the density,  $\propto t_{\text{ff}}^{-2}$ ; and (3) the mass in GMCs,  $\propto f_{\text{GMC}} \Sigma_{\text{g}}$

\* Since  $\Sigma_{\text{g}}/t_{\text{ff}} \propto \rho^{1.5}$ , it shows the nonlinear dependence of the star formation rate on density

\*KM05 showed  $\epsilon_{\text{ff}} \sim$  few percent: **star formation is inefficient**

The star formation efficiency is approximately independent of density  
(Krumholz & Tan 2006)



IRDCs = Infrared Dark Clouds, thought to be a very early stage of star cluster formation

ONC = Orion Nebula Cluster

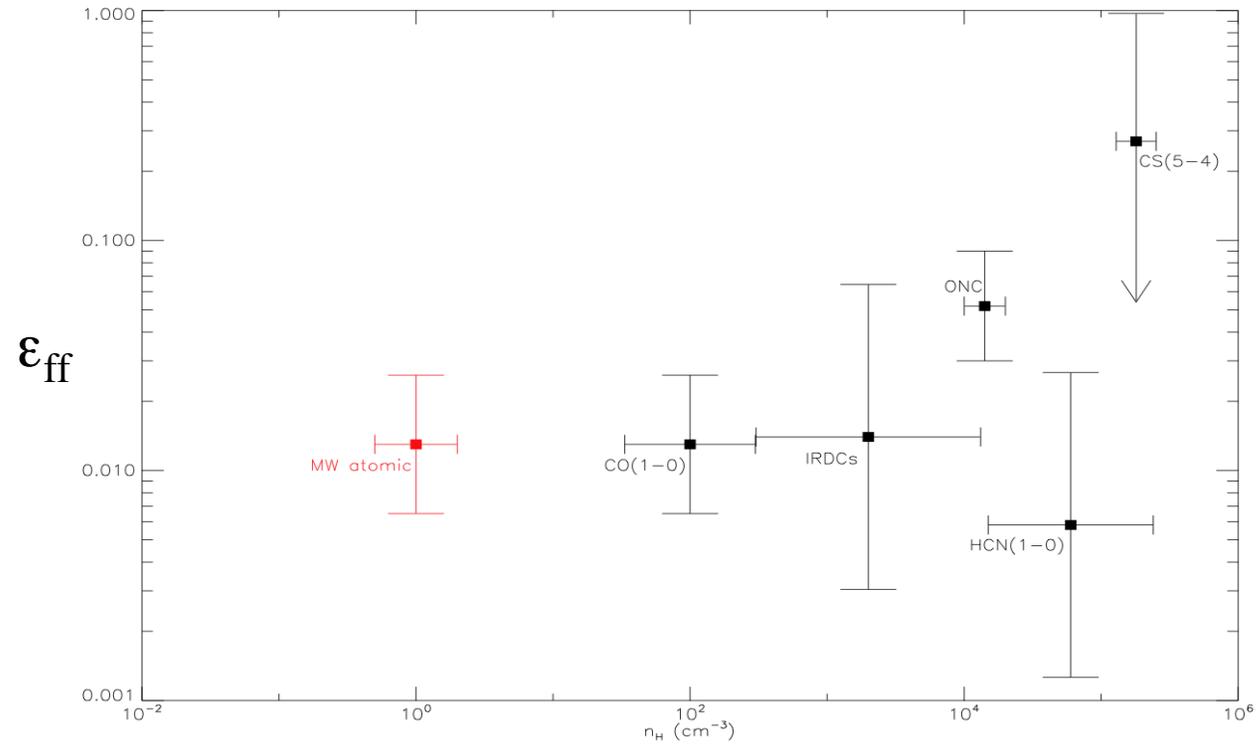
HCN observations of a wide range of galaxies (Gao & Solomon 2004)

CS observations of high-mass star-forming regions in the Galaxy

Survey incomplete  $\Rightarrow$  lower limit on  $M_{\text{CS}} \Rightarrow$  upper limit on  $\epsilon_{\text{ff}}$

Note that the Galaxy has  $\epsilon_{\text{ff}} \approx 1\%$  and Arp 220  $\sim 2\%$

## Global star formation efficiency in the Galaxy $\sim 1\%$

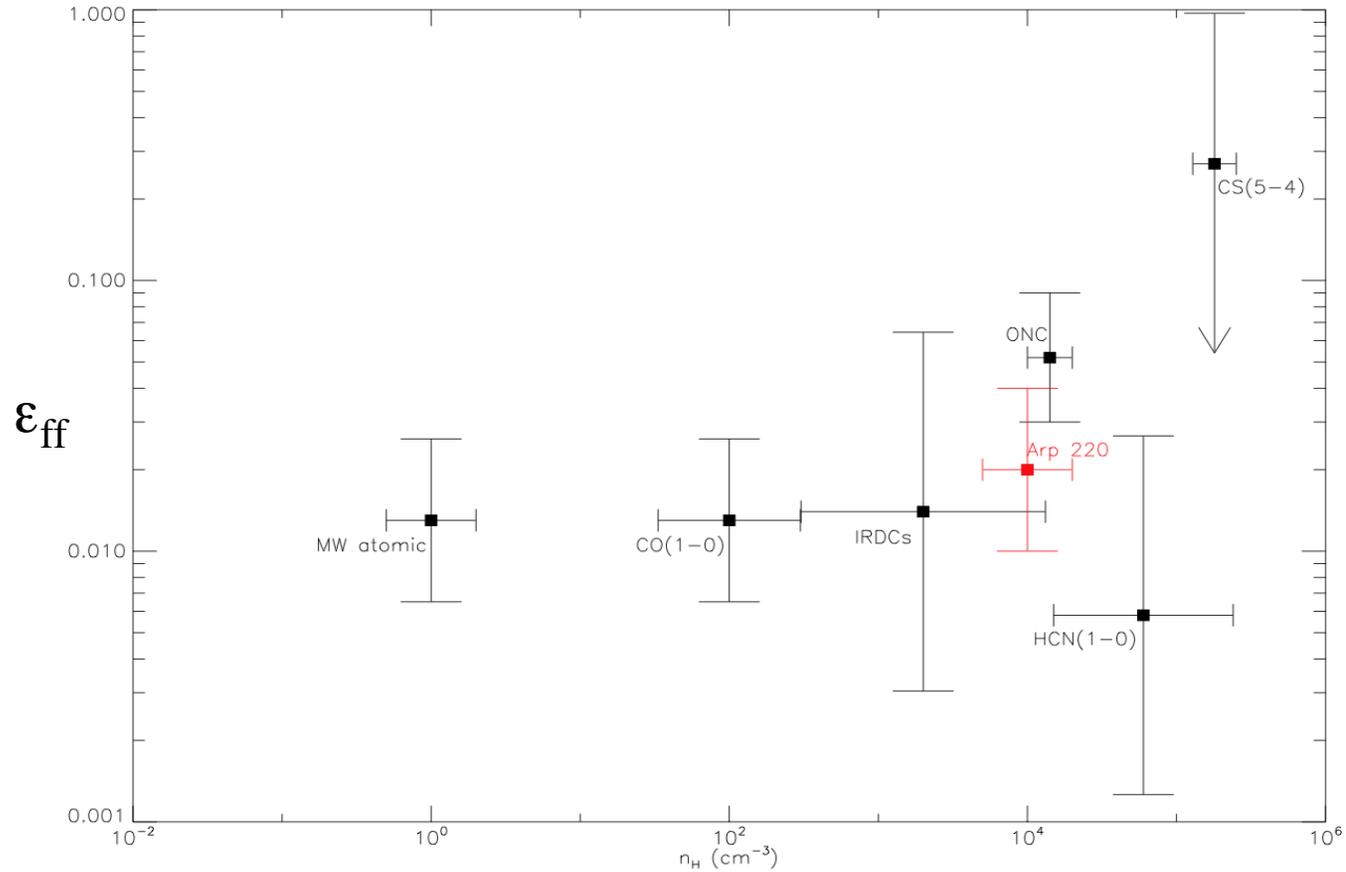


The Milky Way Galaxy:  $\text{SFR} = dM_*/dt \sim 1.5 M_{\text{sun}} \text{ yr}^{-1}$

Atomic gas:  $M_{\text{at}} \sim 5 \times 10^9 M_{\text{sun}} \Rightarrow t_{\text{dep,tot}} \sim 3 \times 10^9 \text{ yr}$

$n \sim 1 \text{ cm}^{-3} \Rightarrow t_{\text{ff}} \sim 4 \times 10^7 \text{ yr} \Rightarrow \epsilon_{\text{ff}} = t_{\text{ff}} / t_{\text{dep,tot}} \sim 0.01$

## Star formation efficiency in Arp 220 (a starburst) $\sim 2\%$



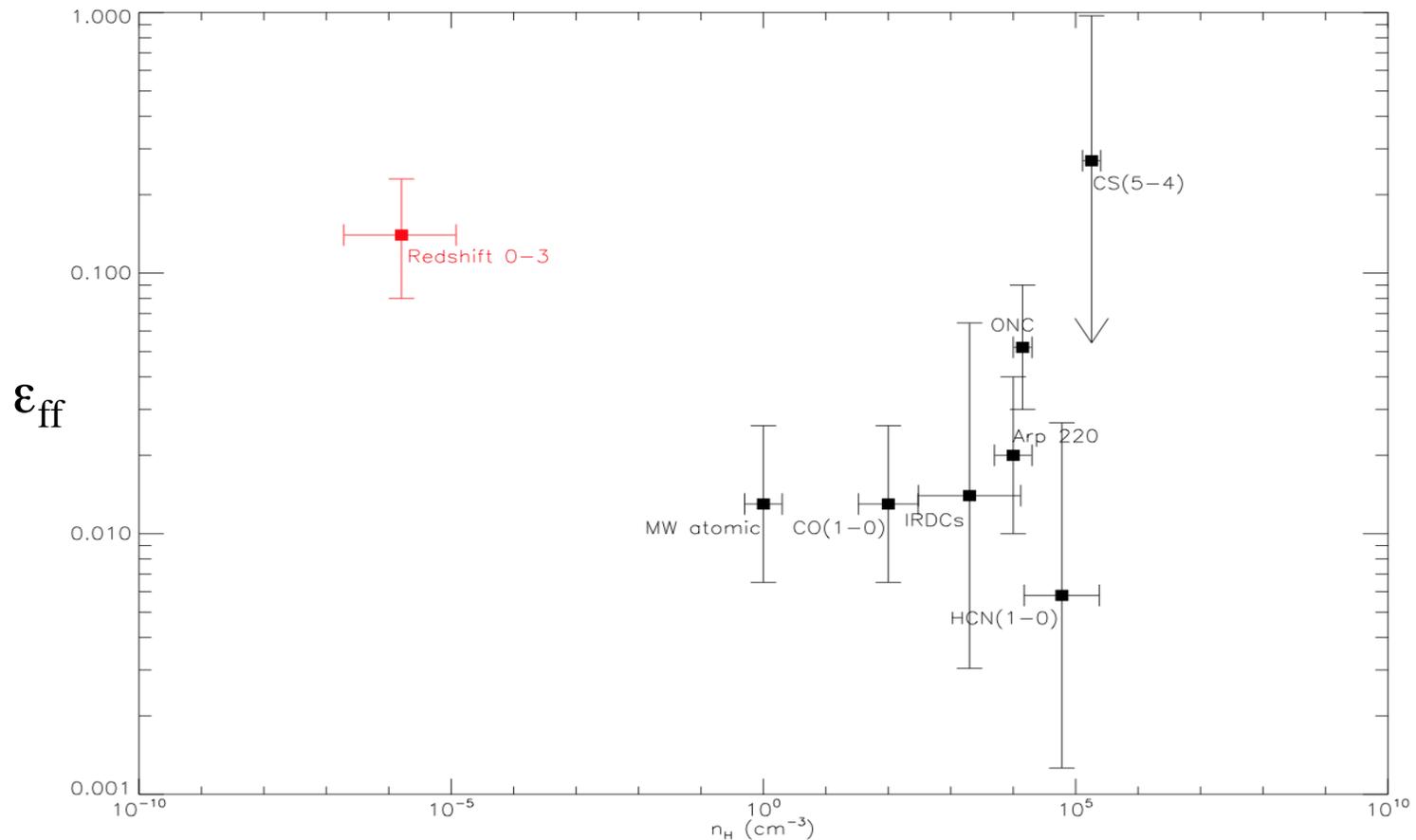
Starburst galaxy Arp 220:  $M_{\text{mol}} \sim 10^9 M_{\text{sun}}$  and  $dM_*/dt \sim 50 M_{\text{sun}} \text{ yr}^{-1}$

$$\Rightarrow t_{\text{dep}} \sim 2 \times 10^7 \text{ yr}$$

The mean density of molecular gas is  $n \sim 10^4 \text{ cm}^{-3} \Rightarrow t_{\text{ff}} \sim 4 \times 10^5 \text{ yr}$

Hence  $\epsilon_{\text{ff}} = t_{\text{ff}} / t_{\text{dep}} \sim 0.02$

Star formation efficiency in the universe is high:  $\epsilon_{\text{ff}} \sim 0.1 - 0.2$



Star formation rate in the universe:  $0.02 (z \sim 0) - 0.1 (z \sim 1-3) M_{\text{sun}}/\text{yr Mpc}^3$   
 Pascale et al 09 (BLAST)

Free-fall time:  $t_{\text{ff}} = 4.2 \times 10^{10} / (1+z)^{3/2} \text{ yr}$  (including dark matter)

$\Rightarrow \epsilon_{\text{ff}} = 0.08 (z=3), 0.13 (z=0), 0.23 (z=1)$

Back to turbulence-regulated star formation in galaxies (KM05):

What is the molecular fraction?

$$\Sigma_{\text{SFR}} = d\Sigma_{*}/dt = (\epsilon_{\text{ff}}/t_{\text{ff}}) f_{\text{GMC}} \Sigma_{\text{g}}$$

What is the Molecular Fraction,  $f(\text{H}_2) \approx f_{\text{GMC}}$  ?

(Krumholz, McKee & Tumlinson 2008, 09;  
McKee & Krumholz 2010)

Strömgren-type analysis:

$F^*$  = incident flux of  
Lyman-Werner photons

$f_{\text{diss}}$  = fraction of absorptions that  
result in dissociation  $\approx 0.1$

Balancing destruction and formation:

$$f_{\text{diss}} F^* = \mathcal{R} n^2 L$$

where  $\mathcal{R} \approx 3 \times 10^{-17} Z' \text{ cm}^3 \text{ s}^{-1}$  is the rate coefficient for  $\text{H}_2$  formation on dust grains

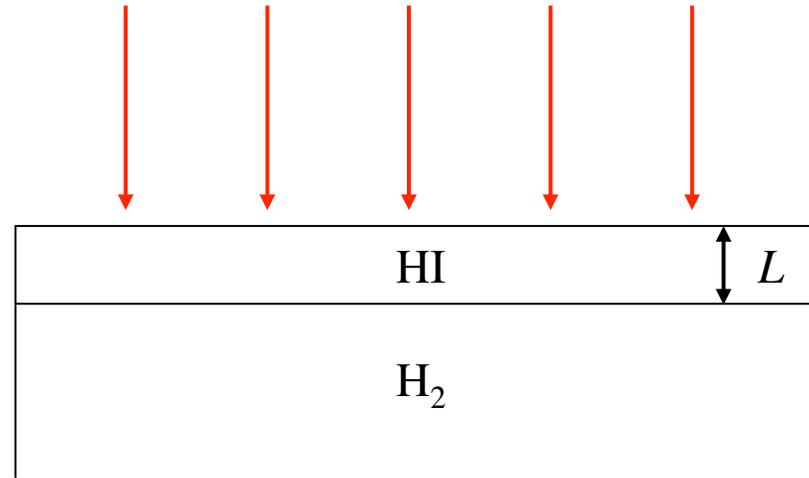
$$\Rightarrow N_{\text{HI}} = nL = f_{\text{diss}} F^* / \mathcal{R} n$$

In two-phase equilibrium,  $n \propto F^*$

Hence surface density of HI shielding layer **is independent of the radiation field & density:**

$$\Sigma_{\text{HI}} \approx 10 / Z' \text{ M}_{\text{sun}} \text{ pc}^{-2}$$

Krumholz & Gnedin (2011) have shown good agreement with models of  
Gnedin & Kravtsov (2011)



# FINAL RESULT FOR TURBULENCE-REGULATED STAR FORMATION:

Predicted star formation rate in galactic disk with total gas surface density  $\Sigma_g$

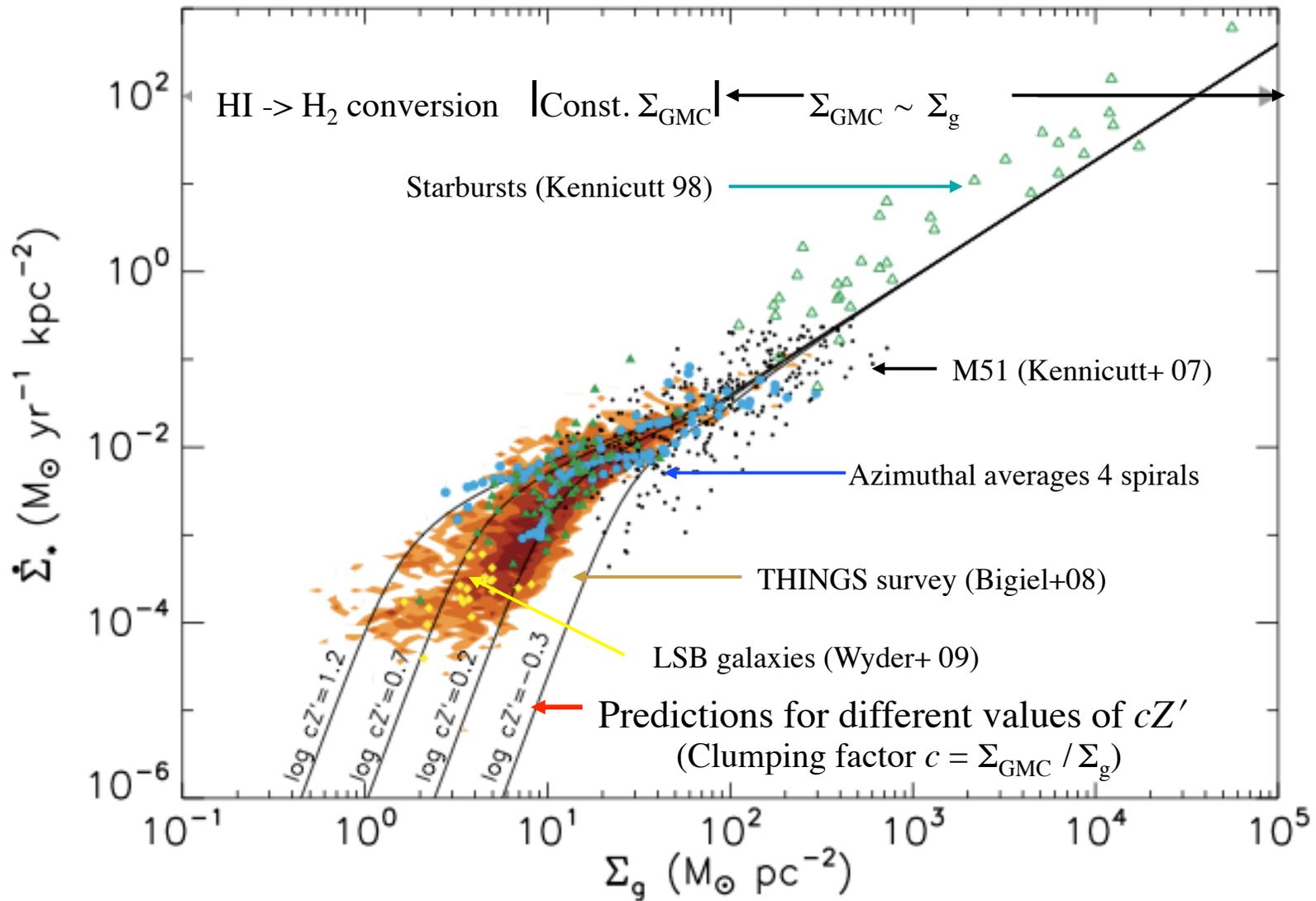
$$\dot{\Sigma}_* = f_{\text{H}_2}(\Sigma_g, c, Z') \frac{\Sigma_g}{2.6 \text{ Gyr}}$$

$$\times \begin{cases} \left( \frac{\Sigma_g}{85 M_\odot \text{ pc}^{-2}} \right)^{-0.33} & , \quad \frac{\Sigma_g}{85 M_\odot \text{ pc}^{-2}} < 1 \\ \left( \frac{\Sigma_g}{85 M_\odot \text{ pc}^{-2}} \right)^{0.33} & , \quad \frac{\Sigma_g}{85 M_\odot \text{ pc}^{-2}} > 1 \end{cases}$$

Three regimes:

- 1)  $\Sigma_g < 10/Z' M_{\text{sun}} \text{ pc}^{-2}$  : HI - H<sub>2</sub> transition
- 2) Normal disks:  $10/Z' M_{\text{sun}} \text{ pc}^{-2} < \Sigma_g < 85 M_{\text{sun}} \text{ pc}^{-2} \sim \Sigma_{\text{GMC}}$
- 3) High surface densities:  $\Sigma_{\text{GMC}} \sim \Sigma_g > 85 M_{\text{sun}} \text{ pc}^{-2}$

Star Formation Rate  $\Sigma_{\text{SFR}}$  vs Total Gas Surface Density  $\Sigma_{\text{g}}$  (KMT 09b)



Excellent statistical agreement between theory and observation

## Summary of Theory I: Turbulence-Regulated Star Formation (Krumholz+)

Approximate, almost first-principles theory of the star formation rate that applies to galaxies with a wide range of surface density (including starbursts) and metallicity

$$\Sigma_{\text{SFR}} = d\Sigma_*/dt = (\epsilon_{\text{ff}}/t_{\text{ff}}) f_{\text{GMC}} \Sigma_{\text{g}}$$

Explicitly predicts inefficiency of star formation,  $\epsilon_{\text{ff}} \sim$  few percent

Explicit calculation of molecular gas fraction,  $f(\text{H}_2)$ ,  $\sim$  gas fraction in GMCs,  $f_{\text{GMC}}$

Two-phase atomic gas  $\Rightarrow f(\text{H}_2)$  depends only on  $\Sigma_{\text{g}}$  and  $Z'$ , not on radiation field

Good statistical agreement between theory and observation

# THEORY II: REGULATION OF STAR FORMATION IN THE TWO-PHASE ISM OF GALACTIC DISKS

(Ostriker, McKee & Leroy 2010)

(Extension: Bolatto et al 2011)

Contrasts with Turbulent Star Formation Theory of Krumholz et al:

- Divide ISM into diffuse gas (2-phase HI) and gravitationally bound clouds (GBCs)  
In principle, GBCs can have substantial mass in atomic envelopes  
(ignored in OML, included in Bolatto et al.)
- Detailed model of two-phase ISM: Prediction of  $\Sigma_{\text{diffuse}}(r)$  for given  $\Sigma_{\text{g}}(r)$
- Includes effect of stellar gravity
- Relies on crucial observational input:
  - Ratio of local SFR to local FUV radiation field
  - Molecular gas depletion time in local galaxies,  $M_{\text{mol}}/(dM_{*}/dt) \approx 2 \text{ Gyr}$
- Applies to normal galaxies ( $\Sigma_{\text{g}} < 100 M_{\text{sun}} \text{ pc}^{-2}$ )
  - OML: Normal metallicity
  - Bolatto: Extension to SMC metallicity ( $Z=0.2$ )

## Relies on three physical principles:

A. Hydrostatic equilibrium of the ISM in the disk:

Midplane pressure = weight of gas above

B. Thermal equilibrium in the ISM: diffuse HI in two phases

Thermal gas pressure  $\propto$  radiation field

C. Star formation equilibrium:

Radiation field  $\propto$  star formation rate  $\propto$  surface density of GBCs

## Approximate solution for star formation rate:

Hydrostatic equilibrium + thermal/star-formation equilibrium imply

$$\Sigma_{\text{SFR}} \simeq \frac{\Sigma_g}{2 \times 10^9 \text{ yr}} \left[ 1 + \frac{9/\phi_D(Z')}{\Sigma_{g,1} + 5.6\rho_{*,-1}^{1/2}} \right]^{-1}$$

where  $\Sigma_{g,1} = \Sigma_g / (10 M_{\text{sun}} \text{ pc}^{-2})$ ,  $\rho_{*,-1} = \rho_* / (0.1 M_{\text{sun}} \text{ pc}^{-3})$ , and  $\phi_D = O(1)$

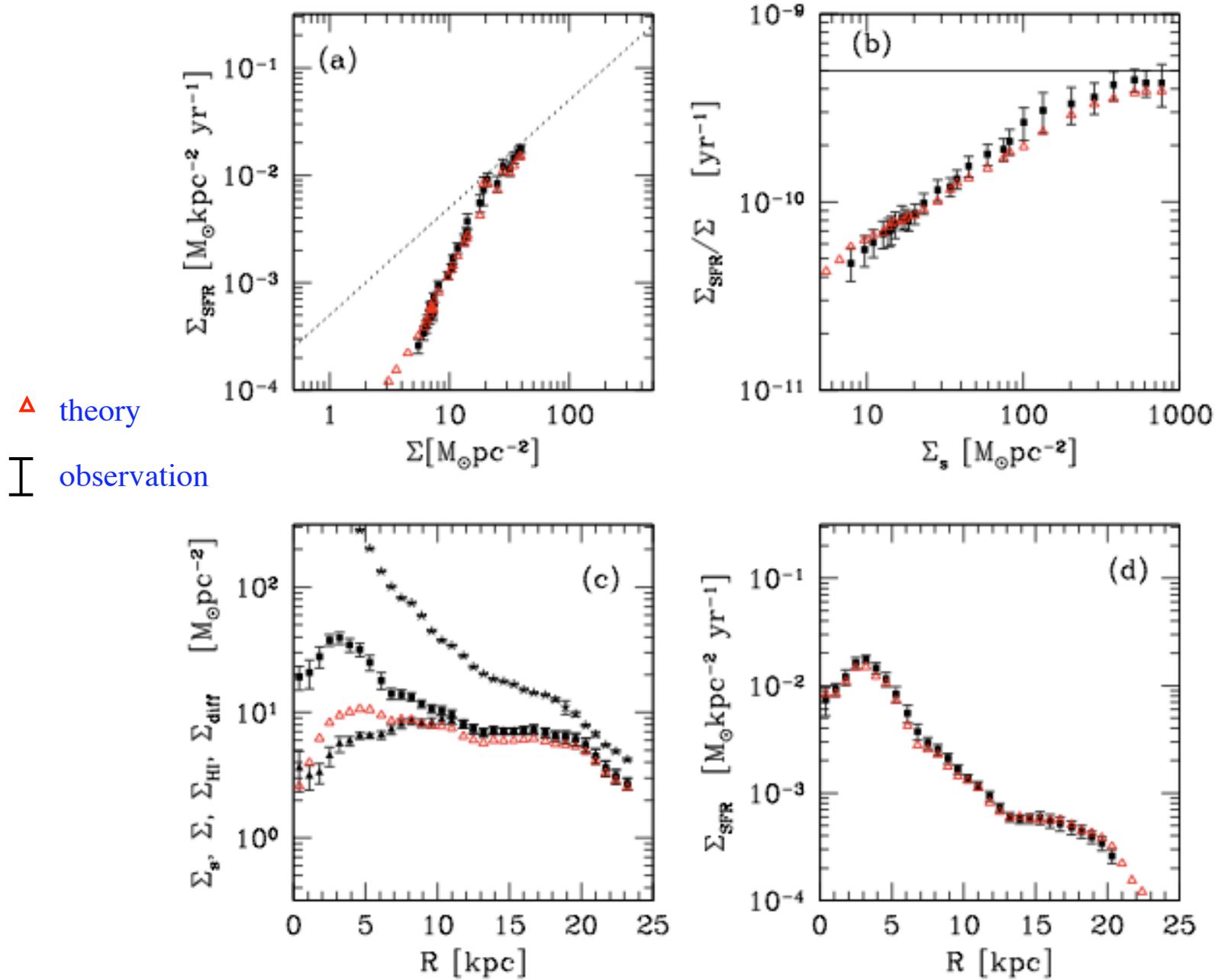
Linear relation between SFR and gas for fully molecular gas:

$$\Sigma_{g,1} + 5.6\rho_{*,-1}^{1/2} \gg 9/\phi_D \Rightarrow \Sigma_{\text{SFR}} = \Sigma_g / (2 \times 10^9 \text{ yr})$$

Compression due to gravity of stars and dark matter generally determines transition from atomic to molecular gas: For  $(\Sigma_{g,1} + 5.6\rho_{*,-1}^{1/2}) < 9/\phi_D$

$$\Sigma_{\text{SFR}} \propto \Sigma_g \rho_*^{1/2} \propto \Sigma_g^a, \text{ with } a > 1$$

Comparison of theory and observation for NGC 7331, a flocculent spiral galaxy

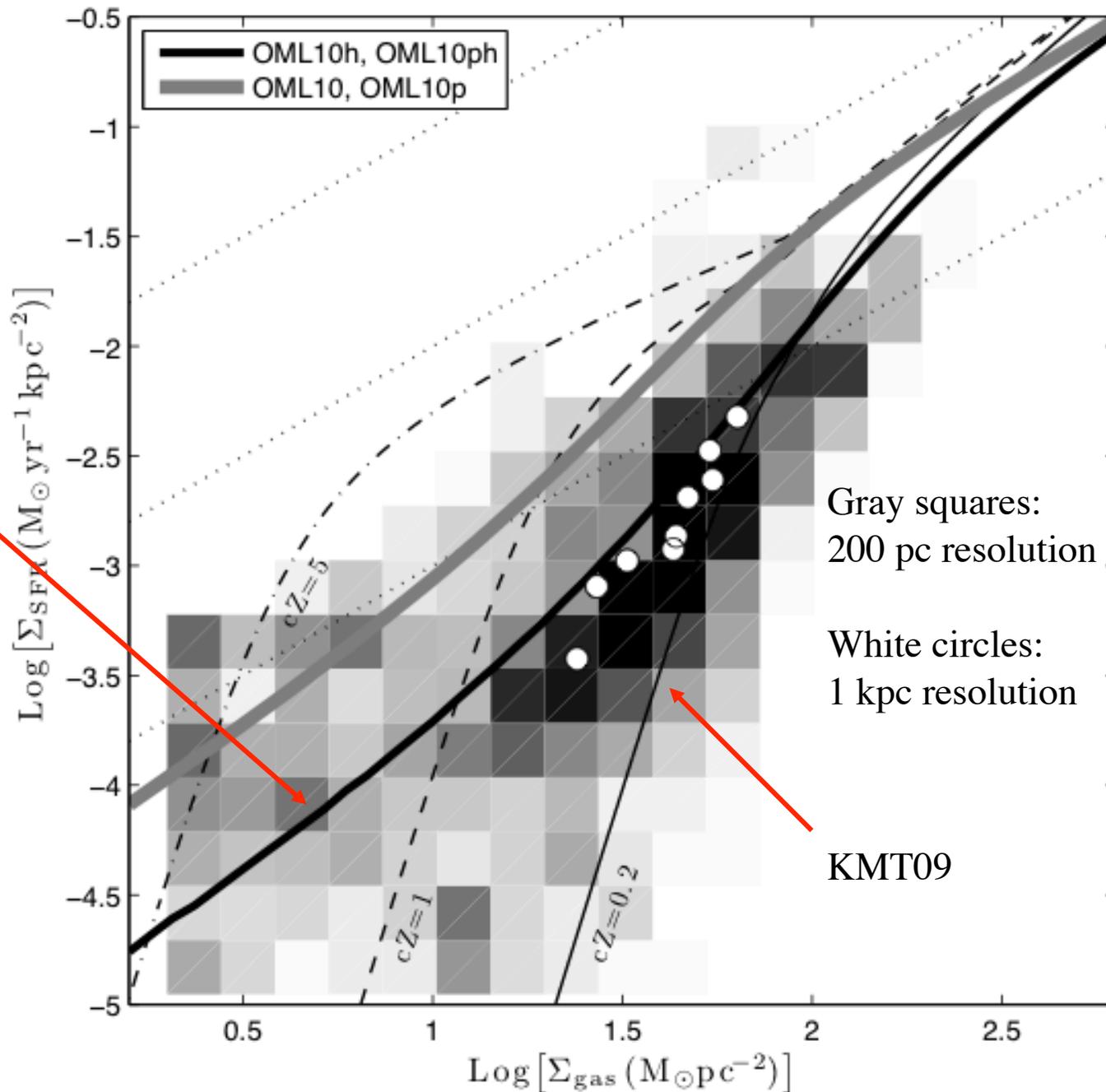


# SMC Star Formation Law

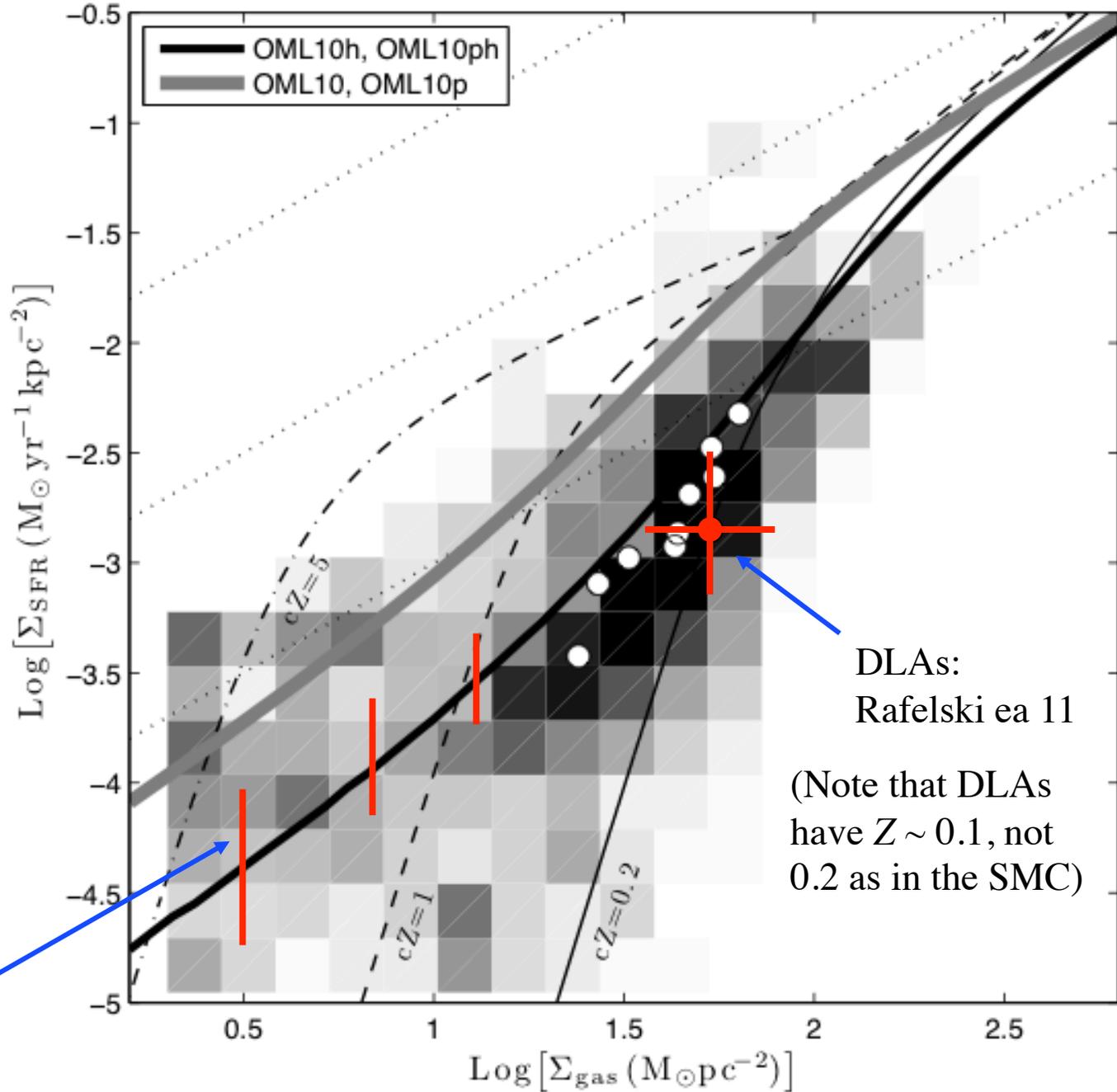
OML theory as revised by Bolatto et al (2011)

Allows for longer FUV mean free path at low Z

Synthesis with KMT planned



Good agreement  
between Bolatto et al  
and observations of  
other systems:



Bigiel ea 2010:  
Outer galaxy disks

DLAs:  
Rafelski ea 11  
  
(Note that DLAs  
have  $Z \sim 0.1$ , not  
 $0.2$  as in the SMC)

# A TALE OF TWO THEORIES OF THE STAR FORMATION RATE: SUMMARY

## II: Regulation of Star Formation in the Two-Phase ISM of Galactic Disks

(Ostriker+ 2010)

More accurate theory for normal galactic disks with moderate metallicity

Key observational input:

Assumes universal star formation time in molecular gas,  $t_{\text{SF}} = 2 \times 10^9$  yr  
and universal ratio of thermal pressure to star formation rate

Physical principles: Hydrostatic equilibrium of galactic disks

(including gravity of stars and dark matter)

+ thermal equilibrium (two-phase ISM),  $P_{\text{th}} \propto G_0'$

+ star formation equilibrium,  $G_0' \propto \Sigma_{\text{SFR}} \propto \Sigma_{\text{GBC}}$

Explicit treatment of diffuse atomic gas and gravitationally bound gas, assumed to be molecular in OML

Excellent agreement between theory and observation of  $\Sigma_{\text{SFR}}$  and  $\Sigma_{\text{HI}}$ , particularly for disk galaxies without grand design spirals

Extension to low  $Z$  for SMC (Bolatto et al) provides good agreement with observations of outer galaxies (Bigiel et al) and DLAs (Rafelski et al)







# A TALE OF TWO THEORIES OF THE STAR FORMATION RATE: SUMMARY

## I: Turbulence-Regulated Star Formation (Krumholz+)

Approximate, almost first-principles theory of the star formation rate that applies to galaxies with a wide range of surface density (including starbursts) and metallicity

$$\Sigma_{\text{SFR}} = d\Sigma_*/dt = (\epsilon_{\text{ff}}/t_{\text{ff}}) f_{\text{GMC}} \Sigma_{\text{g}}$$

Explicitly includes inefficiency of star formation,  $\epsilon_{\text{ff}} \sim$  few percent

Explicit calculation of molecular gas fraction,  $f(\text{H}_2)$ ,  $\sim$  gas fraction in GMCs,  $f_{\text{GMC}}$

Two-phase atomic gas  $\Rightarrow f(\text{H}_2)$  depends only on  $\Sigma_{\text{g}}$  and  $Z'$ , not on radiation field

Does not account for diffuse atomic gas, as in Theory II

Good statistical agreement between theory and observation

## Common Features of Both Theories:

\* Two-phase HI sets the pressure of the diffuse gas in the ISM

Theory I:  $\Rightarrow \Sigma_{\text{HI}} \propto n/G_0'$  is independent of both density and radiation field  
since  $n \propto G_0'$

Theory II:  $P_{\text{th}} \propto G_0'$  is the first step in star-formation equilibrium

\* Star formation rate is a nonlinear function of the surface density of the gas, so theories are less accurate when averaged over regions with strong variations in  $\Sigma_{\text{g}}$

# THEORY OF STAR FORMATION

McKee & Ostriker ARAA 2007

**MICROPHYSICS:** How do individual stars form?

**MACROPHYSICS:** How do systems of stars, from clusters to galaxies, form?

- What determines the amount of molecular gas (the fuel for star formation) in a galaxy?
- What determines the rate at which stars form?
- What determines the Initial Mass Function (IMF) of stars?
- How do stars form in clusters?

# THE STAR FORMATION RATE IN GALAXIES: OBSERVATION

Global SFR:

Schmidt (1959, 1963) proposed that the Star Formation Rate (SFR)  $\propto$  (density)<sup>*n*</sup> or (surface density)<sup>*n*</sup>, with  $n > 1$

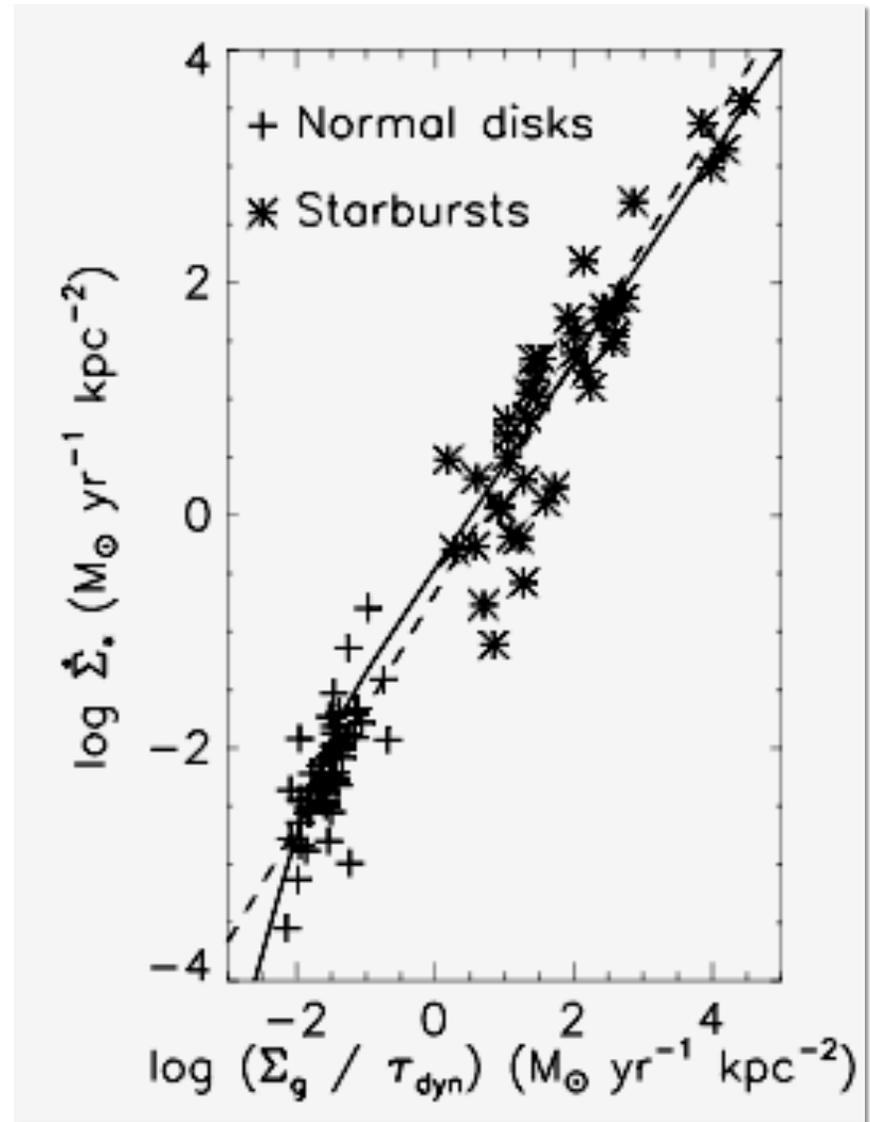
Kennicutt (e.g., 1998) focused on disk galaxies and starbursts and found two forms for the SFR per unit area:

$$\begin{aligned} d\Sigma_*/dt &= 0.16 \Sigma_{g,2}^{1.4} & M_{\text{sun}} \text{ yr}^{-1} \text{ kpc}^{-2} & \quad \text{where } \Sigma_{g,2} = \Sigma_g / (100 M_{\text{sun}} \text{ pc}^{-2}) \\ &= 0.017 \Sigma_g \Omega & M_{\text{sun}} \text{ yr}^{-1} \text{ kpc}^{-2} & \quad \text{where } \Omega = \text{rotation rate of outer disk} \end{aligned}$$

Data for the star formation law for normal disks and starbursts

(Kennicutt 1998)

Subsequent observations have shown that the SFR is related to the surface density of *molecular* gas (Wong & Blitz 2002; Kennicutt+ 2007).



$$\tau_{\text{dyn}} = 4\pi / \Omega$$

- - - Power-law fit (Kennicutt 98)

Chris McKee  
User:

P<sub>h</sub> is Leroy+  
fit based on  
entire sample

### Spatially Resolved SFRs:

### Radial Profiles of Stars, Gas and SFR in NGC 7331

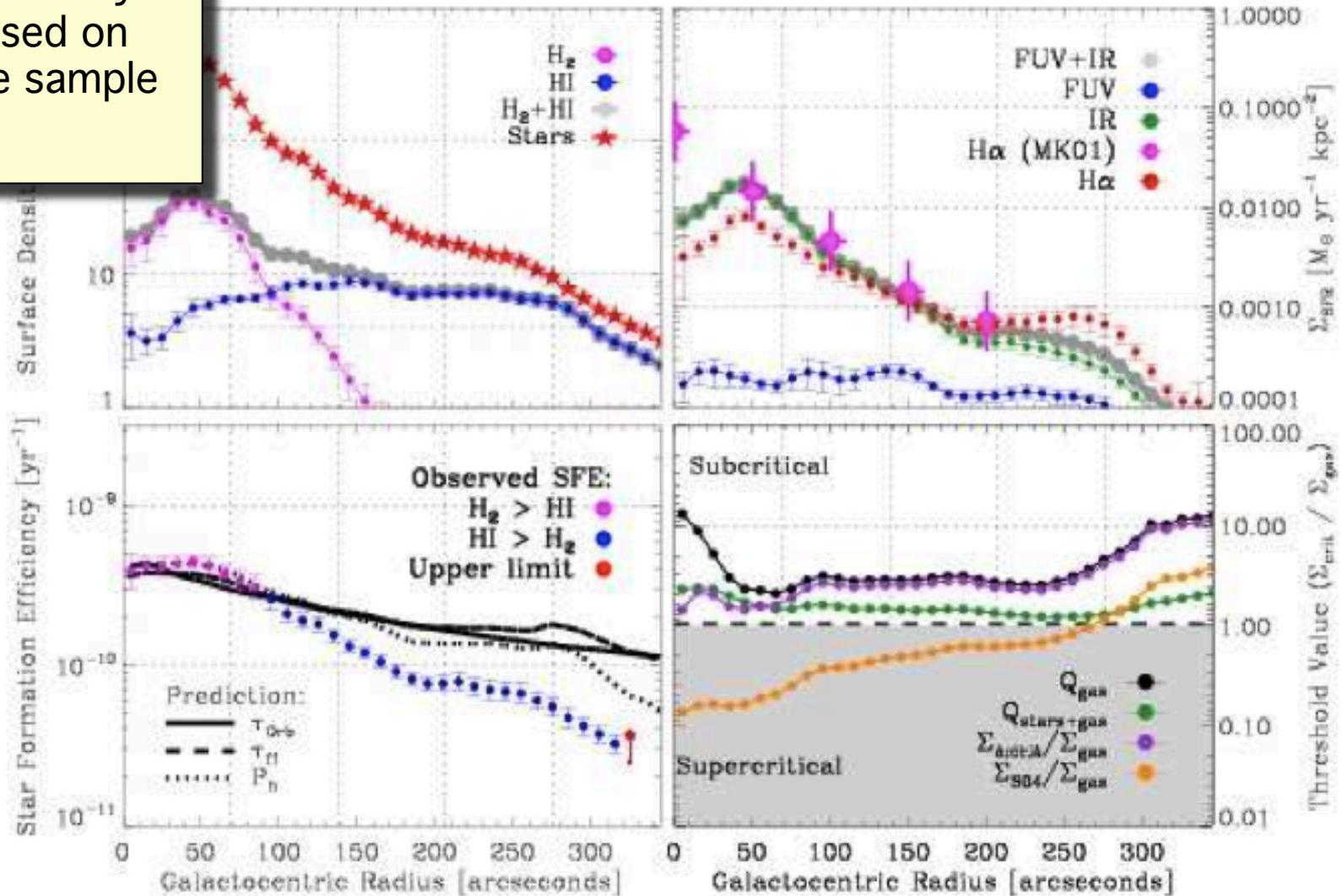
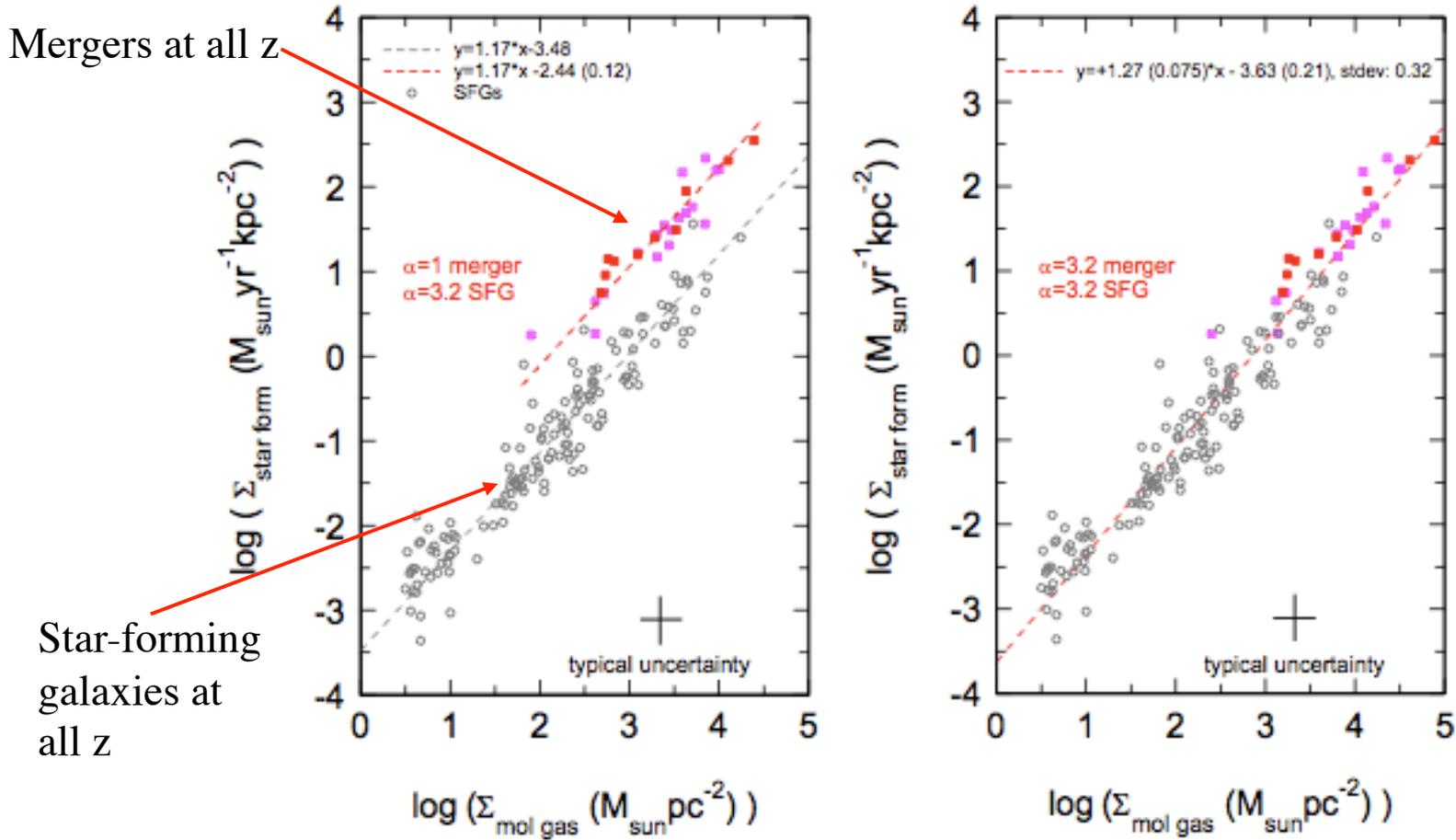


FIG. F.— Atlas of data and calculations for NGC 7331.

(Leroy et al. 2008)

Does the star formation law vary with redshift?

How do mergers affect the star formation rate?



(Genzel+ 2010)

$$\Sigma_{\text{SFR}} = d\Sigma_{*}/dt = (\epsilon_{\text{ff}}/t_{\text{ff}}) f_{\text{GMC}} \Sigma_{\text{g}}$$

## 1. STAR FORMATION IS INEFFICIENT: $\epsilon_{\text{ff}} \sim$ FEW PERCENT (Zuckerman & Evans 1974)

Define the depletion time as the time to convert the molecular gas into stars:

$$t_{\text{dep}} \equiv M_{\text{mol}} / (dM_{*}/dt) \quad (\text{global})$$

$$\equiv \Sigma_{\text{mol}} / (d\Sigma_{*}/dt) = f_{\text{GMC}} \Sigma_{\text{g}} / (d\Sigma_{*}/dt) \quad (\text{local})$$

since most molecular gas in GMCs

Then  $d\Sigma_{*}/dt = \epsilon_{\text{ff}} f_{\text{GMC}} \Sigma_{\text{g}} / t_{\text{ff}} \Rightarrow \epsilon_{\text{ff}} = t_{\text{ff}} / t_{\text{dep}}$

The Milky Way Galaxy:  $M_{\text{mol}} \sim 10^9 M_{\text{sun}}$  and  $\text{SFR} = dM_{*}/dt \sim 3 M_{\text{sun}} \text{ yr}^{-1}$

$$\Rightarrow t_{\text{dep}} \sim 3 \times 10^8 \text{ yr}$$

The mean density of GMCs is  $n \sim 10^2 \text{ cm}^{-3} \Rightarrow t_{\text{ff}} \sim 4 \times 10^6 \text{ yr}$

Hence  $\epsilon_{\text{ff}} = t_{\text{ff}} / t_{\text{dep}} \sim 0.01$

$$\Sigma_{\text{SFR}} = d\Sigma_{*}/dt = (\epsilon_{\text{ff}}/t_{\text{ff}}) f_{\text{GMC}} \Sigma_{\text{g}}$$

## 1. STAR FORMATION IS INEFFICIENT: $\epsilon_{\text{ff}} \sim$ FEW PERCENT (Zuckerman & Evans 1974)

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Then  $d\Sigma_{*}/dt = \epsilon_{\text{ff}} f_{\text{GMC}} \Sigma_{\text{g}} / t_{\text{ff}} \Rightarrow \epsilon_{\text{ff}} = t_{\text{ff}} / t_{\text{dep}}$

Starburst galaxy Arp 220:  $M_{\text{mol}} \sim 10^9 M_{\text{sun}}$  and  $dM_{*}/dt \sim 50 M_{\text{sun}} \text{ yr}^{-1}$

$$\Rightarrow t_{\text{dep}} \sim 2 \times 10^7 \text{ yr}$$

The mean density of molecular gas is  $n \sim 10^4 \text{ cm}^{-3} \Rightarrow t_{\text{ff}} \sim 4 \times 10^5 \text{ yr}$

Hence  $\epsilon_{\text{ff}} = t_{\text{ff}} / t_{\text{dep}} \sim 0.02$

## Star Formation Efficiency in a Turbulent Medium

Turbulent gas has a log-normal density distribution that depends only on the Mach number

$$p(\rho) \propto \exp [ - ( \ln \rho - \langle \ln \rho \rangle )^2 / 2 \sigma_\rho^2 ]$$

where  $\sigma_\rho^2 \approx \ln ( 1 + 0.75 \mathcal{M}^2 )$

$\mathcal{M} = \sigma/c_s$  is the Mach number

(Padoan & Nordlund 2002)

Only high-density regions with a virial parameter

$$\alpha_{\text{vir}}(r) = 5\sigma^2 R / GM \sim 2\text{KE}/\text{PE} \sim 1$$

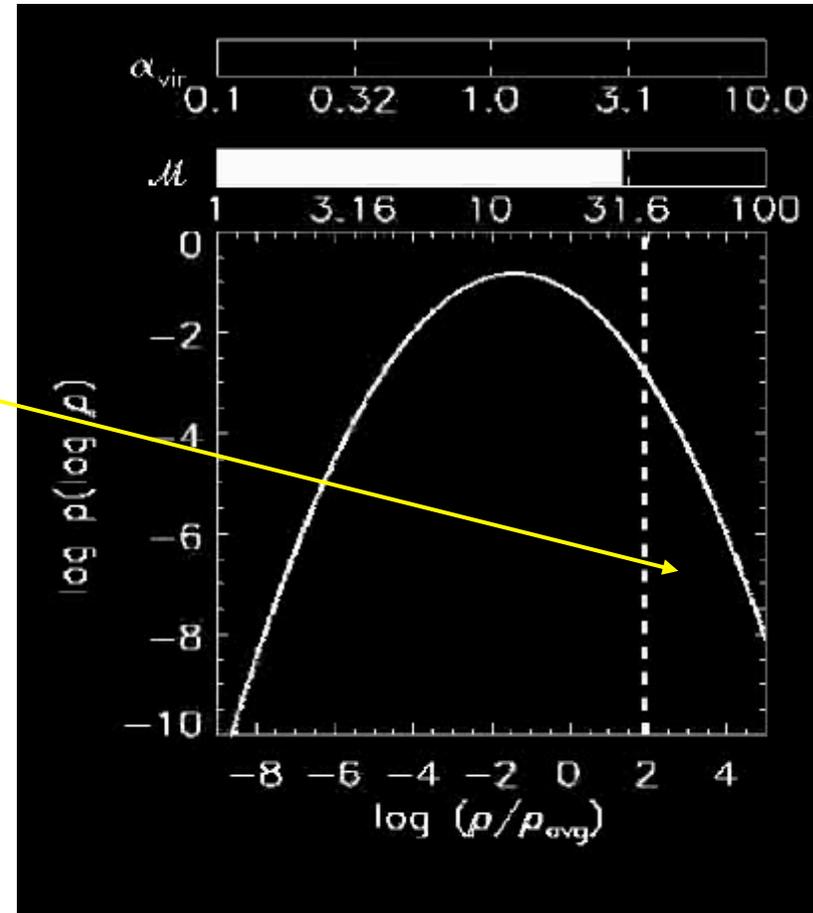
can collapse, and these regions form on the free-fall time of the GMC.

Result is an estimate for the star formation efficiency per free-fall time:

$$\epsilon_{\text{ff}} \approx 0.017 \alpha_{\text{vir}}^{-0.84} \Sigma_{\text{cl},2}^{-0.08} M_6^{-0.08}$$

which is a few percent for all turbulent, virialized objects.

Here,  $\Sigma_{\text{cl},2} = \text{cloud surface density} / (100 M_{\text{sun}} \text{ pc}^{-2})$  and  $M_6 = \text{cloud mass} / (10^6 M_{\text{sun}})$



$$\Sigma_{\text{SFR}} = d\Sigma_{*}/dt = (\epsilon_{\text{ff}}/t_{\text{ff}}) f_{\text{GMC}} \Sigma_{\text{g}}$$

## 2. Evaluation of free-fall time $t_{\text{ff}}$ and GMC surface density $\Sigma_{\text{GMC}}$

$$t_{\text{ff}} = (3\pi/32 G\rho)^{1/2}$$

$$\rho \propto (\Sigma_{\text{GMC}}^3 / M_{\text{GMC}})^{1/2}$$

What is  $\Sigma_{\text{GMC}}$  ?

Theory based on time-dependent virial theorem suggests  $\Sigma_{\text{GMC}} \sim 100 M_{\text{sun}} \text{ pc}^{-2}$   
(Krumholz, Matzner & McKee 2006)

Observations of normal galaxies consistent with theory:

Local group galaxies have  $\Sigma_{\text{GMC}} \sim 85 M_{\text{sun}} \text{ pc}^{-2}$  (Blitz+ 07, Bolatto+ 08)

Heyer+ 09 have revised the masses of Galactic GMCs down from

$\Sigma_{\text{GMC}} \sim 170 M_{\text{sun}} \text{ pc}^{-2}$  of Solomon+ 87 and are consistent with  $85 M_{\text{sun}} \text{ pc}^{-2}$

Starbursts: For  $\Sigma_{\text{g}} > 85 M_{\text{sun}} \text{ pc}^{-2}$ , assume  $\Sigma_{\text{GMC}} \sim \Sigma_{\text{g}}$

$$\Rightarrow t_{\text{ff}} \approx 8 M_6^{1/4} \min [ 1, (85 M_{\text{sun}} \text{ pc}^{-2} / \Sigma_{\text{GMC}})^{3/4} ] \text{ Myr}$$

Infer  $M_{\text{GMC}}$  from Jeans mass in galactic disk with a Toomre  $Q$  parameter =1

## Molecular fraction in a two-phase medium (KMT 2009a)

Recall density of HI in two-phase equilibrium:

$$n_{2\text{phase}} \approx 20 G_0' / \phi_D \text{ cm}^{-3}$$

where  $G_0'$  = FUV radiation/local ISM value

$$\phi_D = (1 + 3.1 Z'^{0.365}) / 4.1$$

$Z'$  = metallicity relative to solar

Stromgren analysis  $\Rightarrow \Sigma_{\text{HI}} \propto G_0' / n Z'$

Hence surface density of HI shielding layer **is independent of the radiation field:**

$$\Sigma_{\text{HI}} \approx 10 (\phi_D / Z') M_{\text{sun}} \text{ pc}^{-2}$$

Molecular fraction  $f(\text{H}_2) = 1 - \Sigma_{\text{HI}} / \Sigma_g$

Assume  $f_{\text{GMC}} \approx f(\text{H}_2)$

# Generalization to spherical clouds with isotropic incident radiation

(McKee & Krumholz 2010)

First two moments of equation of radiative transfer:

$$\frac{1}{r^2} \frac{d}{dr} r^2 F^* = c(\kappa_E + \kappa_d) E^*,$$
$$\frac{dP^*}{dr} + \frac{3P^* - E^*}{r} = \frac{1}{c} (\kappa_F + \kappa_d) F^*,$$

where  $E^*$  is the number density of LW photons

$F^*$  is the flux of LW photons

$P^*$  is the  $rr$  component of the radiation pressure tensor for LW photons

$\kappa_{E,F}$  is the frequency-integrated line opacity, weighted by  $E_v^*$  and  $F_v^*$

$\kappa_d$  is the dust opacity

Solve using assumed form for  $P^*/E^*$  that allows for shadowing by cloud core  $H_2$

Analytic fit to numerical solution for molecular fraction of atomic-molecular complex:

$$f(H_2) \approx 1 - 0.75 s / (1 + 0.25 s)$$

where  $s = \ln [\text{fct}(Z)] / (\text{dust optical depth of complex})$

## Comparison with Observation:

Recall the molecular fraction of an atomic-molecular complex:

$$f(\text{H}_2) \approx 1 - 0.75 s / (1 + 0.25 s)$$

where  $s = \ln [\text{fct} (Z')] / (\text{dust optical depth of complex})$

Dust optical depth of complex  $\propto \sigma_d \Sigma_{\text{complex}}$

where the dust cross section  $\sigma_d \approx 10^{-21} Z' \text{ cm}^2$

Observations of gas surface density  $\Sigma_g$  generally cannot resolve the complexes

Hence, introduce the clumping factor  $c = \Sigma_{\text{complex}} / \Sigma_g$ , typically  $\sim 5$

Then  $f(\text{H}_2)$  depends primarily on the dust optical depth  $\propto \sigma_d \Sigma_{\text{complex}} \propto c Z' \Sigma_g$

## Existing Observational Model for the Star Formation Rate

Wong & Blitz 2002; Blitz & Rosolowsky 2004, 06; Leroy+ 2008)

Empirical relation between the ratio of molecular to atomic gas and the estimated midplane pressure

$$\Sigma_{\text{mol}}/\Sigma_{\text{atom}} \propto P_{\text{mid}}^{\gamma} \propto (\Sigma_{\text{g}} \rho_{\text{star}}^{1/2})^{\gamma}, \quad \text{with } \gamma \approx 0.8 - 0.9$$

+ the observed relation between the star formation rate and molecular gas in normal spirals

$$\Sigma_{\text{SFR}} = \Sigma_{\text{mol}}/t_{\text{SF}} \quad \text{with } t_{\text{SF}} \sim 2 \times 10^9 \text{ yr}$$

gives reasonably good fit to radial dependence of the star formation rate,  $\Sigma_{\text{SFR}}(r)$

Leroy+ 08 improved this fit by including the self-gravity of the gas

The physical justification for the relation between molecular fraction and the midplane pressure is unclear; furthermore, the effect of dark matter on the weight of the gas (and therefore its pressure) is not taken into account

## A. Hydrostatic equilibrium: Midplane pressure set by weight of ISM

$$dP/dz = -\rho_{\text{diff}} g \quad \Rightarrow \quad dP/d\Sigma_{\text{diff}} \sim -g, \text{ where } \Sigma_{\text{diff}} \text{ is the diffuse gas mass / area}$$

Recall electrostatics: For a sheet with  $\sigma = \text{charge/area}$ ,  $E = 2\pi\sigma$

$$\text{For a disk galaxy, } g = -2\pi G \Sigma_{\text{total}} = -2\pi G ( \Sigma_{\text{diff}} + \Sigma_{\text{gbc}} + \rho_* H_{\text{diff}} )$$

$$\Rightarrow P \sim \pi G ( \Sigma_{\text{diff}}^2/2 + \Sigma_{\text{diff}} \Sigma_{\text{gbc}} + \Sigma_{\text{diff}} \rho_* H_{\text{diff}} )$$

$H_{\text{diff}} \propto \Sigma_{\text{diff}} / P_{\text{th}}$  is the scale height of diffuse gas

\* Total gas pressure = Thermal pressure  $P_{\text{th}}$  + turbulent pressure + net magnetic pressure:  $P = \alpha P_{\text{th}}$

\* Self-gravity of diffuse gas

\* Weight of diffuse gas in field of GBCs (assumed to have small scale height)

\* Weight of diffuse gas in field of stars and dark matter;  $H_{\text{diff}} = \text{scale height}$   
Dominant term over most of disk in normal galaxies

Generalization of results of Elmegreen (1989) and Blitz & Rosolowsky (2004, 06)

$\Rightarrow$  Hydrostatic equilibrium gives first relation between  $\Sigma_{\text{gbc}} = \Sigma_{\text{g}} - \Sigma_{\text{diff}}$  and  $P_{\text{th}}$

## B. Thermal equilibrium: Two-phase ISM $\Rightarrow P_{\text{th}}(G_0')$

$$\text{Recall } P_{\text{th}}/k = P_{2\text{phase}}/k \approx 3000 G_0'/\phi_D \quad \text{K cm}^{-3}$$

where  $G_0' = \text{FUV radiation/local ISM value}$

$$\phi_D = (1 + 3.1 Z'^{0.365})/4.1$$

$Z' = \text{metallicity relative to solar}$

## C. Star-formation equilibrium gives second relation between $\Sigma_{\text{gbc}}$ and $P_{\text{th}}$ :

1. Radiation field  $G_0'$  is proportional to the star formation rate:

$$G_0' \equiv \frac{J_{\text{FUV}}}{J_{\text{FUV},0}} = \frac{\Sigma_{\text{SFR}}}{\Sigma_{\text{SFR},0}} = \frac{\Sigma_{\text{SFR}}}{2.5 \times 10^{-9} \text{ M}_{\odot} \text{ pc}^{-2} \text{ yr}^{-1}}$$

consistent with theoretical estimates

2. Express star formation rate in terms of  $\Sigma_{\text{gbc}}$

$$\Sigma_{\text{SFR}} = \Sigma_{\text{gbc}} / t_{\text{SF}}$$

where  $t_{\text{SF}} = 2 \times 10^9 \text{ yr}$  is the star formation time in molecular gas in local disks

$$\Rightarrow P_{\text{th}}/k = 600 \Sigma_{\text{gbc}} / \phi_D \quad \text{K cm}^{-3} : \text{assumed universal}$$

## Stability of star-formation equilibrium

a) Diffuse gas fraction too high  $\Rightarrow$  high pressure

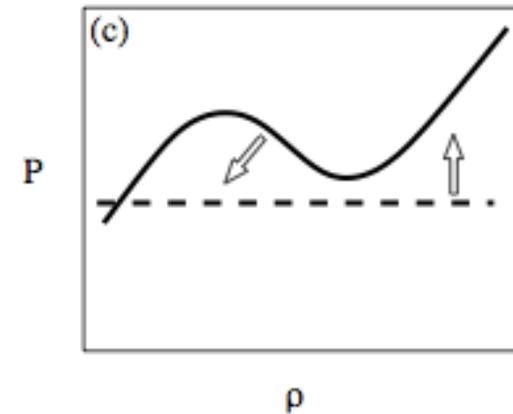
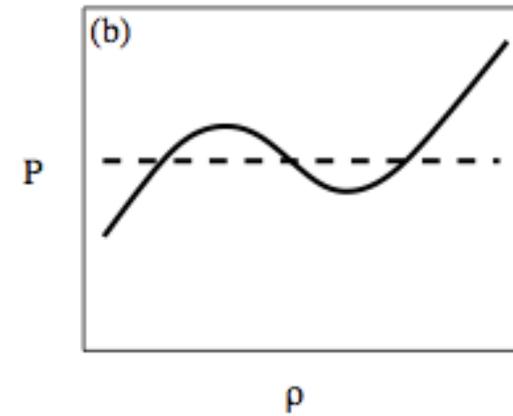
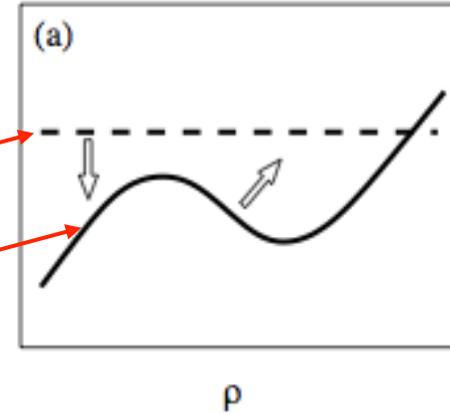
Also, GBC fraction too low  $\Rightarrow$  low SFR and two-phase pressure

Low SFR allows diffuse gas to become bound, lowering the pressure and increasing the SFR & two-phase pressure  $\Rightarrow$  state (b)

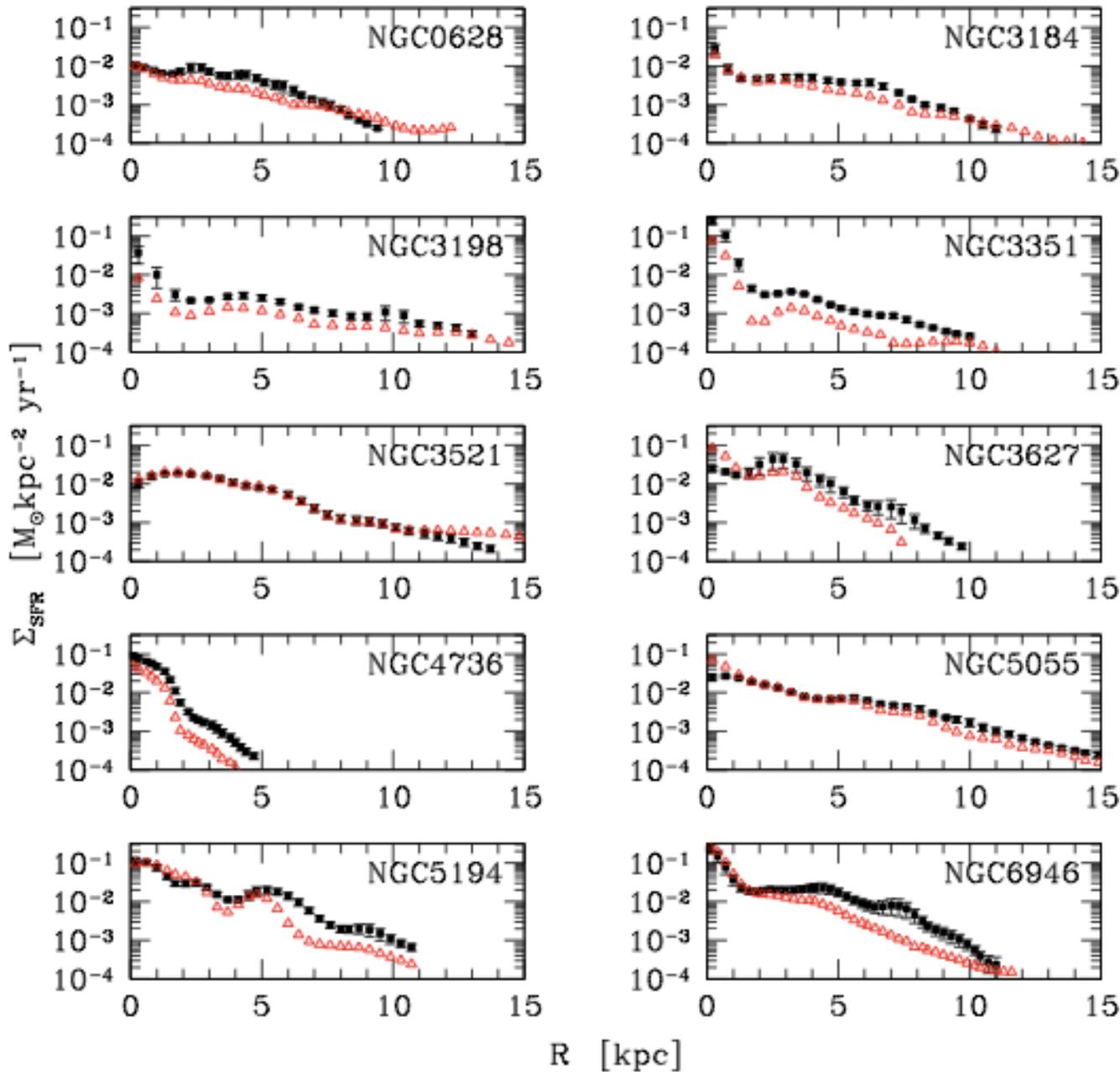
c) Diffuse gas fraction too small  $\Rightarrow$  low pressure

Also, GBC fraction too high  $\Rightarrow$  high SFR and two-phase pressure

High SFR transfers gas from GBCs to diffuse state, raising the pressure and decreasing the SFR & two-phase pressure  $\Rightarrow$  state (b)



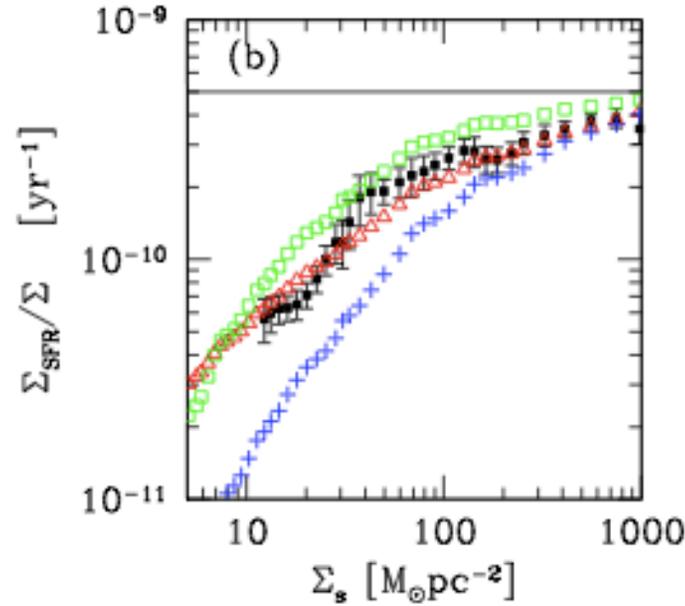
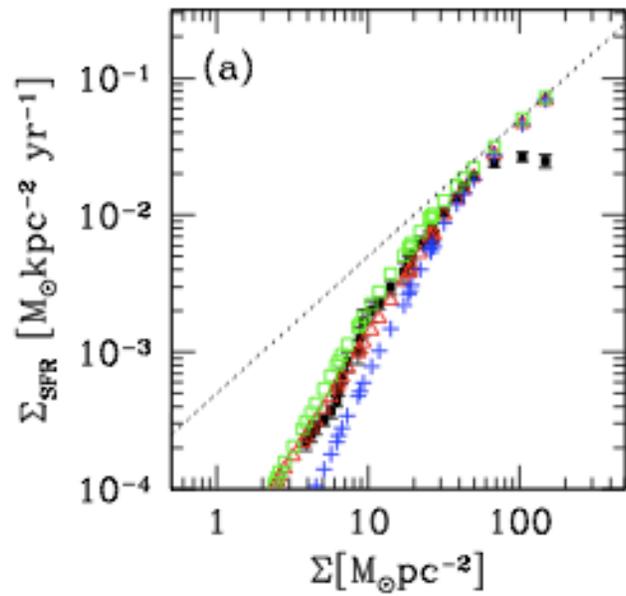
## Comparison between theory and observation for 10 spiral galaxies



Azimuthal averages do not take into account nonlinearities due to spiral density waves.

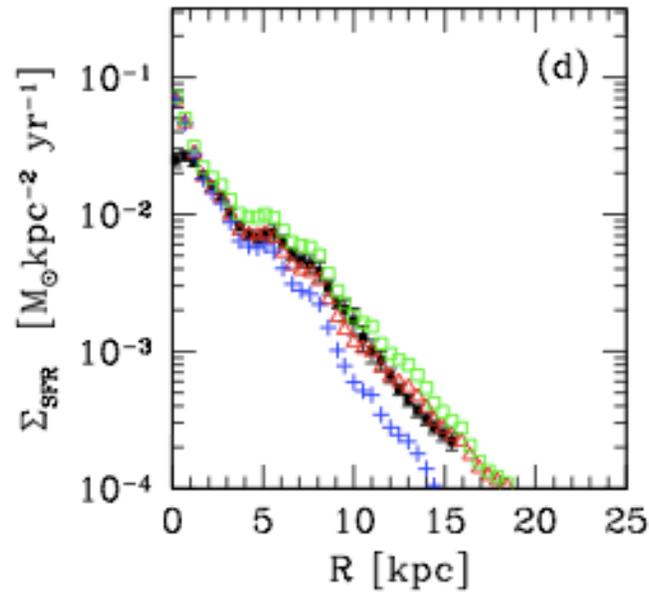
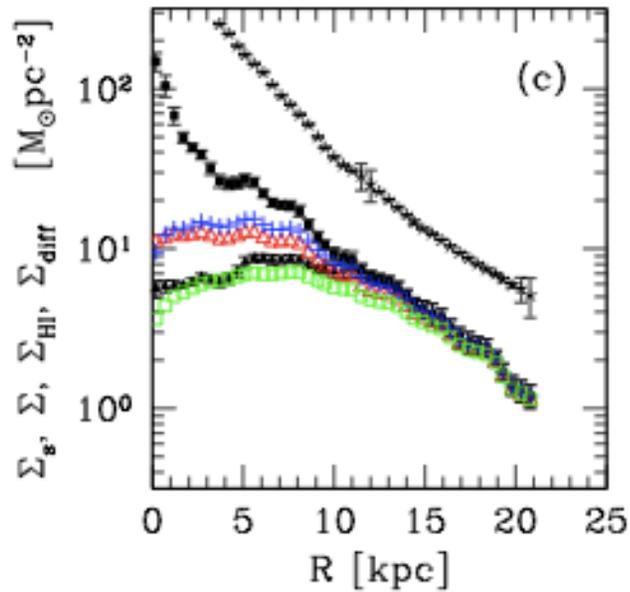
The fits could be improved by varying  $f_w/\alpha$ , flaring of the stellar disk, the dark matter profile, and the dust to gas ratio.

# Comparison of theory, observational models, and observation for NGC 5055



- + Blitz+R 06
- ▲ Leroy+ 08

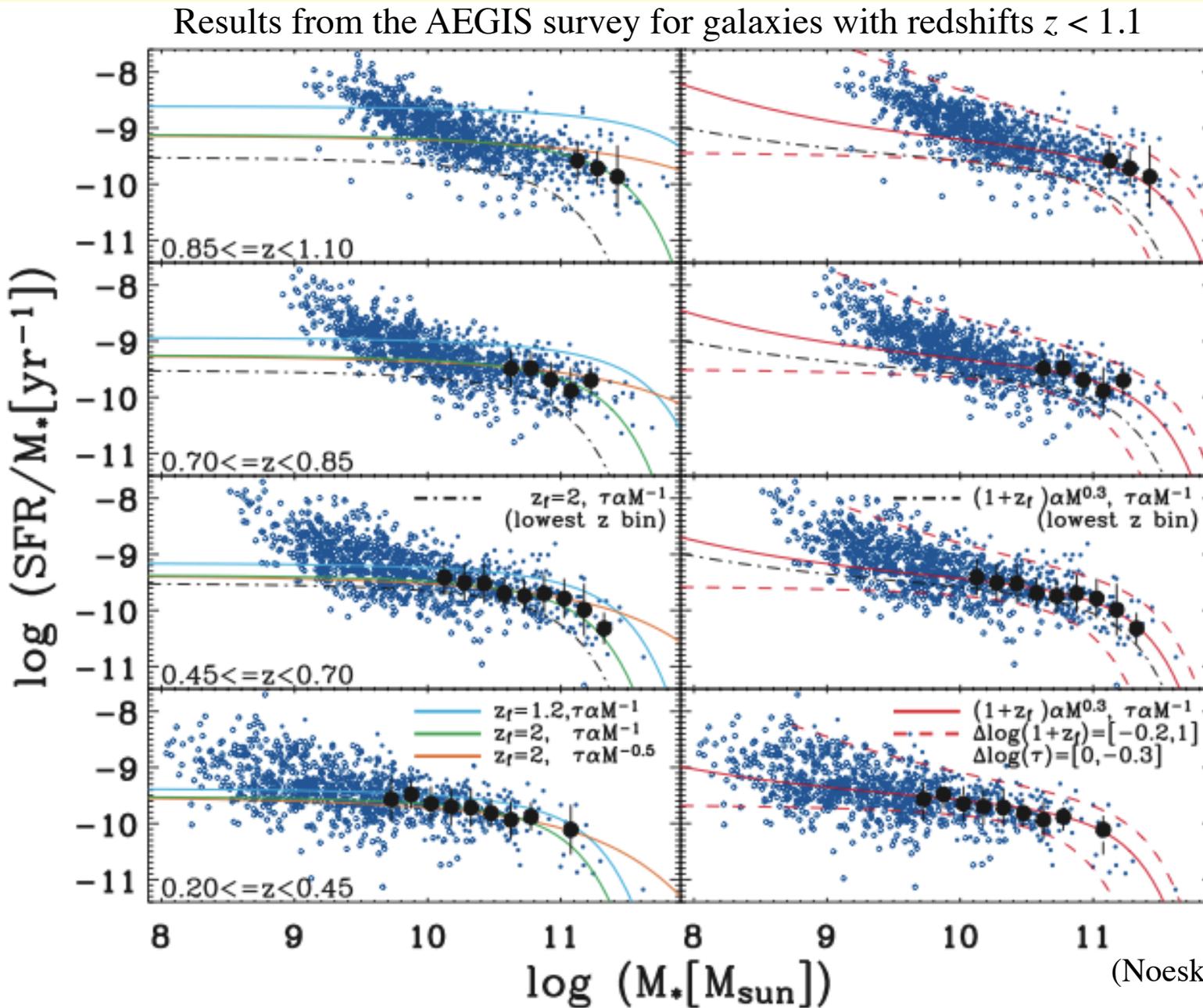
For this case only, we used fitted metallicity profiles (Dutil & Roy 99)



Chris McKee User:

What determines the star formation rate in galaxies as they evolve?

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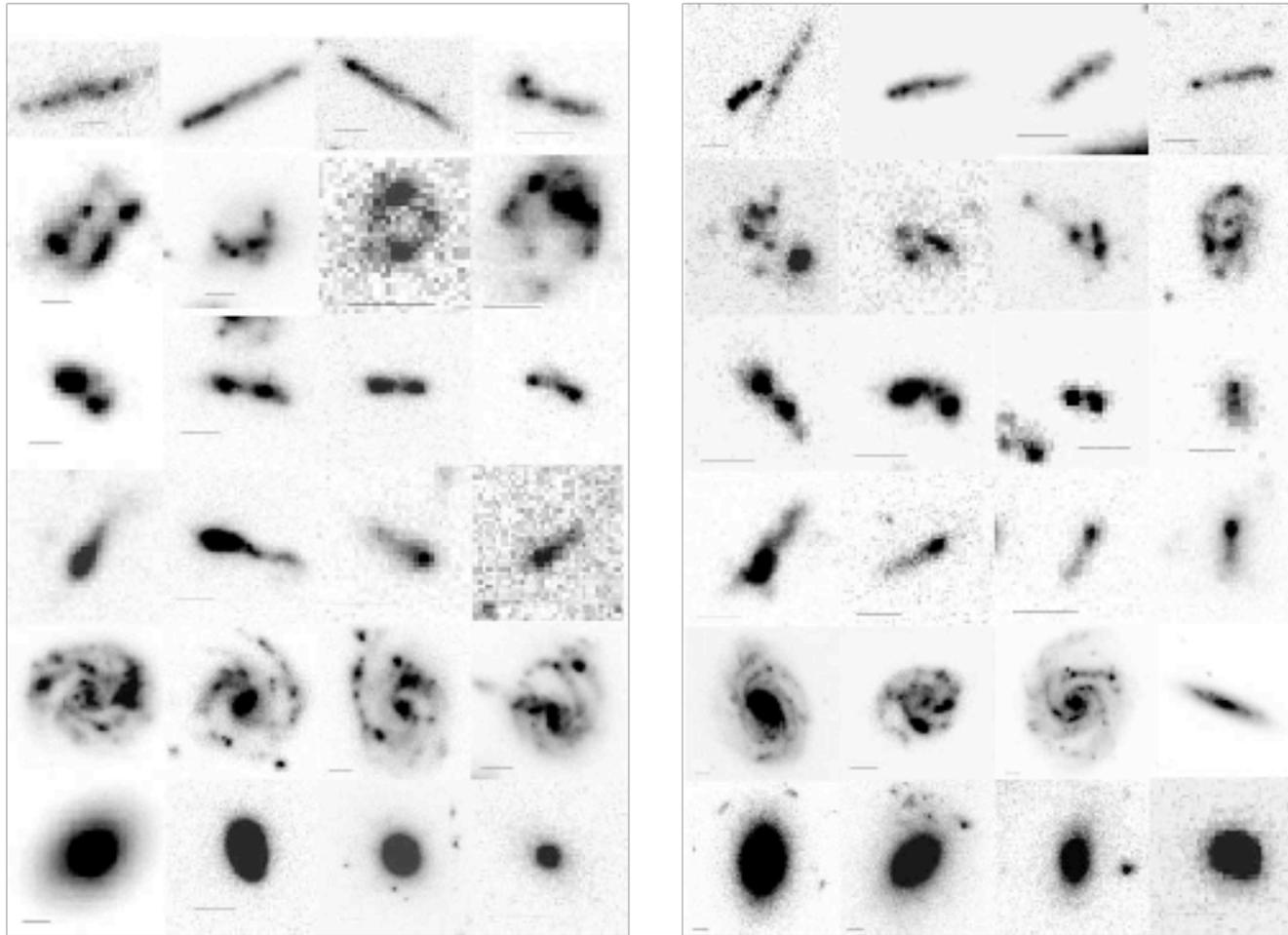


FIG. 1a

FIG. 1a

FIG. 1b

FIG. 1.— Selection of eight typical galaxies for each morphological type: four in (a) and four in (b). *Top to bottom*: Chain, clump-cluster, double, tadpole, spiral, and elliptical galaxies. Images are at  $i_{775}$  band, with a line representing  $0''.5$ . UDF or our own identification numbers from left to right in (a) are as follows: chains: 6478, 7269, 6922, 3214; clump clusters: CC12, 1375, 3291, 5190; doubles: 637, 4072, 5098, 5251; tadpoles: 3058, 8614, 5358, 6891; spirals: 3372, 3180, 4438, 8275; ellipticals: 2107, 4380, 3322, 0911. In (b), the identifications are: chains: 169 and 110 (two separate galaxies), 1423, 401, 3459+5413; clump cluster: 6285, 0907, 7110, 9159; doubles: 2461, 2558, 4097, 3967; tadpoles: 9543, 5115, 3147, 9348; spirals: 2607, 5805, 7556, 5670; ellipticals: 8, 4527, 4320, 5959. Panel b has an example of an edge-on spiral.

tadpole (97), spiral (269), and elliptical (100). Figure 1 shows eight examples of each type; the lines correspond to  $0''.5$ .

Galaxy morphology can vary with wavelength, so we viewed many of the cataloged objects at other ACS passbands and with

*Tadpole*.—Systems dominated by a single clump that is off-center from, or at the end of, a more diffuse linear emission.

*Spiral*.—Galaxies with exponential-like disks, evident spiral structure if they have low inclination, and usually a bulge or a

# THE CURRENT PARADIGM FOR STAR FORMATION: STARS FORM IN SUPERSONICALLY TURBULENT GAS

Turbulence introduces a new length scale, the **sonic length**  $\lambda_s$ :

Velocity dispersion in box of size  $l$ :  $\sigma = c_s(l/\lambda_s)^q$  where  $c_s$  = isothermal sound speed

For Kolmogorov turbulence,  $q = 1/3$  and  $\lambda_s \gg l_{\max}$

For supersonic turbulence,  $q \approx 1/2$  and  $\lambda_s < l_{\max}$

In Galactic molecular clouds  $\lambda_s \approx 0.14$  pc, and it appears to be about constant

(see Heyer & Brunt 2004)

Low-mass stars form from thermally supported cores with a mass  $\sim$  Jeans mass

Thermal support  $\Rightarrow \lambda_s > R$

Gravitationally unstable  $\Rightarrow R > \sim \lambda_J$  where  $\lambda_J \sim c_s/(G\rho)^{1/2}$  = Jeans length

Hence a necessary condition for low-mass star formation is that the density be high enough that  $\lambda_s > \lambda_J$  (Padoan 1995): turbulence must be small on the scale of the thermal Jeans length.

[However, high-mass stars form in turbulent gas,  $R > \lambda_s$ ]

(McKee & Tan 2002, 03; Hennebelle & Chabrier 2008, 09)

## A. Hydrostatic equilibrium: Midplane pressure set by weight of ISM

$$P_{\text{th}} \left( 1 + \frac{v_t^2}{c_w^2 \tilde{f}_w} \right) = \frac{\pi G}{2} \Sigma_{\text{diff}}^2 + \pi G \Sigma_{\text{gbc}} \Sigma_{\text{diff}} + 2\pi \zeta_d G c_w^2 \tilde{f}_w \frac{\rho_* \Sigma_{\text{diff}}^2}{P_{\text{th}}}$$

\* Total gas pressure: Thermal pressure  $P_{\text{th}} = \rho_w c_w^2 \approx f_w \rho_0 c_w^2$  (where  $f_w = \text{WNM mass fraction}$ ) + turbulent pressure + net magnetic pressure

\* Self-gravity of diffuse gas

\* Weight of diffuse gas in field of GBCs (assumed to have small scale height)

\* Weight of diffuse gas in field of stars and dark matter;  $\zeta_d \approx 0.33$

Dominant term over most of disk in normal galaxies

Generalization of results of Elmegreen (1989) and Blitz & Rosolowsky (2004, 06)

Total midplane gas pressure is  $\alpha P_{\text{th}}$ , where

$$\alpha \equiv 1 + \frac{v_t^2}{c_w^2 \tilde{f}_w} = \frac{\langle v_{\text{th}}^2 \rangle + v_t^2}{\langle v_{\text{th}}^2 \rangle}$$

## Approximate solution for star formation rate:

Hydrostatic equilibrium + thermal/star-formation equilibrium imply

$$\Sigma_{\text{SFR}} \simeq \frac{\Sigma_g}{2 \times 10^9 \text{ yr}} \left[ 1 + \frac{9/\phi_D(Z')}{\Sigma_{g,1} + 5.6\rho_{*, -1}^{1/2}} \right]^{-1}$$

where  $\Sigma_{g,1} = \Sigma/(10 M_{\text{sun}} \text{ pc}^{-2})$  and  $\rho_{*, -1} = \rho_*/(0.1 M_{\text{sun}} \text{ pc}^{-3})$

Linear relation between SFR and gas for fully molecular gas in normal galaxies ( $\Sigma_g < 100 M_{\text{sun}} \text{ pc}^{-2}$ )

Compression due to gravity of stars and dark matter generally determines transition from atomic to molecular gas:

$$\Sigma_{\text{SFR}} \propto \Sigma_g \rho_*^{1/2} \propto \Sigma_g^a, \text{ with } a > 1$$

Gas becomes fully molecular for  $\Sigma_{g,1}$  and/or  $\rho_{*, -1} \gg 1$

⇒ Hydrostatic equilibrium gives first relation between  $f_{\text{gbc}} = \Sigma_{\text{gbc}} / \Sigma_{\text{g}}$  and  $P_{\text{th}}$ :

$$P_{\text{th}} \left( 1 + \frac{v_t^2}{c_w^2 \tilde{f}_w} \right) = \frac{\pi G}{2} \Sigma_{\text{diff}}^2 + \pi G \Sigma_{\text{gbc}} \Sigma_{\text{diff}} + 2\pi \zeta_d G c_w^2 \tilde{f}_w \frac{\rho_* \Sigma_{\text{diff}}^2}{P_{\text{th}}}$$

$$\Rightarrow P_{\text{th}} = \frac{\pi G \Sigma_{\text{g}}^2}{2\alpha} \left[ (1 - f_{\text{gbc}})^2 + 2f_{\text{gbc}}(1 - f_{\text{gbc}}) + \left( \frac{\pi G \Sigma_{\text{g}}^2}{2\alpha P_{\text{th}}} \right) S (1 - f_{\text{gbc}})^2 \right]$$

where 
$$S \equiv \frac{8\zeta_d c_w^2 \tilde{f}_w \alpha \rho_*}{\pi G \Sigma_{\text{g}}^2} = 16 (\rho_* / 0.05 \text{ M}_{\text{sun}} \text{ pc}^{-3})(10 \text{ M}_{\text{sun}} \text{ pc}^{-2} / \Sigma_{\text{g}})^2$$

measures the importance of stellar + dm gravity ( $S \approx 16$  near Sun)

Fixed parameters: take turbulent parameter  $\alpha \sim 5$ , WNM mass fraction  $f_w \sim 1/2$

Take  $\Sigma_{\text{g}}(r)$  and  $\Sigma_{\text{star}}(r)$  from observation

Infer  $\rho_{\text{star}}(r) = \Sigma_{\text{star}}(r) / (0.54 R)$ , where  $R$  is the radial stellar scale height (Leroy+ 08)

Infer  $\rho_{\text{dm}}(r)$  from rotation curve; then  $\rho_*(r) = \rho_{\text{star}}(r) + \rho_{\text{dm}}(r)$  determines  $S$

Insert result from star formation equilibrium,

$$P_{\text{th}}/k = 600 \Sigma_{\text{gbc}} / \phi_{\text{D}} \text{ K cm}^{-3} \quad \text{with } \Sigma_{\text{gbc}} = f_{\text{gbc}} \Sigma_{\text{g}}$$

into result from hydrostatic equilibrium,

$$P_{\text{th}} = \frac{\pi G \Sigma_{\text{g}}^2}{2\alpha} \left[ (1 - f_{\text{gbc}})^2 + 2f_{\text{gbc}}(1 - f_{\text{gbc}}) + \left( \frac{\pi G \Sigma_{\text{g}}^2}{2\alpha P_{\text{th}}} \right) S(1 - f_{\text{gbc}})^2 \right].$$

Then solve resulting cubic for  $f_{\text{gbc}}$ , which implies the star formation rate,

$$\Sigma_{\text{SFR}} = f_{\text{gbc}} \Sigma_{\text{g}} / (2 \times 10^9 \text{ yr}) .$$

## The Next Steps:

Observational--Test theories in regions of extreme conditions:

- Low metallicity dwarf galaxies
- Regions of high stellar density
- Starbursts
- Outer parts of galaxies

Theoretical--Synthesize the two theories by incorporating the methods of I into II:

- Calculate the molecular content of Gravitationally Bound Clouds (GBCs)  
-----important for low metallicity
- Calculate the star formation time in molecular gas,  $t_{\text{SF}} \sim 2 \times 10^9$  yr, as a function of physical conditions
- Calculate the relation between the UV radiation field and the star formation rate,  $G_0' \propto \Sigma_{\text{SFR}}$

